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MATHEMATICS 14

Module 5
GEOMETRY

Learning Technologies Branch

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MATHEMATICS 14

Module 5

GEOMETRY

Mathematics 14
Module 5: Geometry
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

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- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome to

MATHEMATICS 14

Mathematics 14 contains five modules. You should work through the modules in order (from 1 to 5) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1
NUMBER

Module 2
PATTERNS and EQUATIONS

Module 3
FRACTIONS, RATIO, and PERCENT

Module 4
MEASUREMENT

Module 5
GEOMETRY

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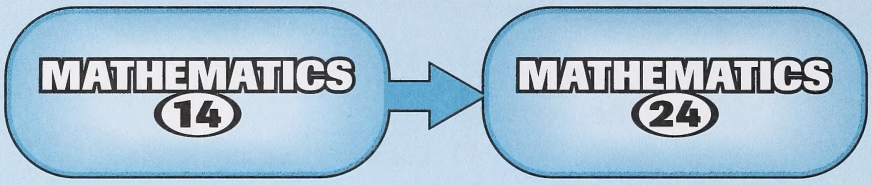
COURSE FEATURES

The Mathematics 14–24 Program

Mathematics 14 is the first course in the Mathematics 14–24 sequence of courses. If you successfully complete each of these five-credit courses, you will meet the minimum requirements in mathematics for an Alberta high school diploma.



The Mathematics 14–24 sequence is designed for students whose needs, interests, and abilities focus on basic mathematical understanding. This course sequence emphasizes the acquisition of practical life skills and proficiency in using mathematics to solve problems, adapt to change, interpret information, and build on previous knowledge.

Consult your teacher or counsellor for the latest information. Also, if you have access to the Internet, you can find out more about Mathematics 14 and high school requirements at the Alberta Learning website.

http://www.learning.gov.ab.ca

Take the time to look through the Student Module Booklets and the Assignment Booklets and notice the following design features:

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several sections. Each section focuses on a big idea that is central to the topic being learned in the module.
- Each section has several lessons.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module references the CD that accompanies your *Continuum* textbook.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

In order to complete the course, you will need a copy of the Mathematics 14 textbook, *Continuum*, a scientific calculator (such as the Texas Instruments TI-30X IIS), and various manipulatives (pattern blocks and fraction blocks). For your convenience, cut-out fraction blocks are provided in the Appendix of Module 3.

Pattern blocks and fraction blocks are available from the Learning Resources Centre. As of 2003, the product codes for these items were 161901 and 408288, respectively. Check for the latest ordering information at the LRC website.

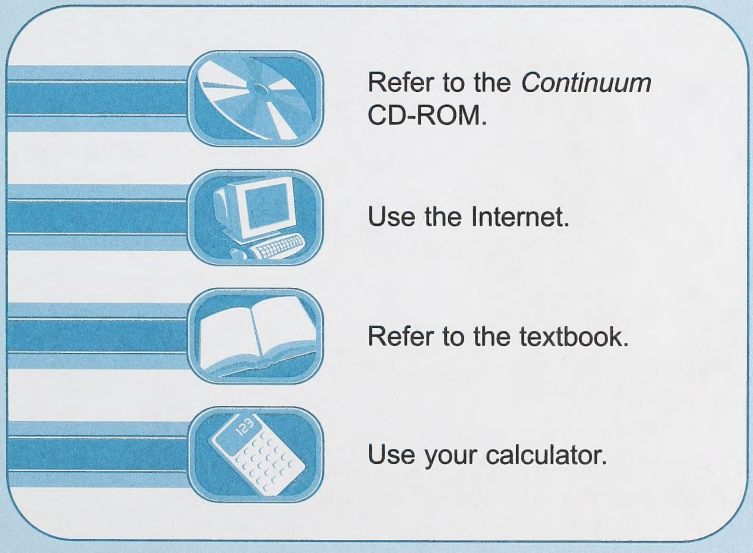
http://www.lrc.learning.gov.ab.ca

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

You will also need access to a computer to view material on the CD-ROM that accompanies your *Continuum* textbook.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.

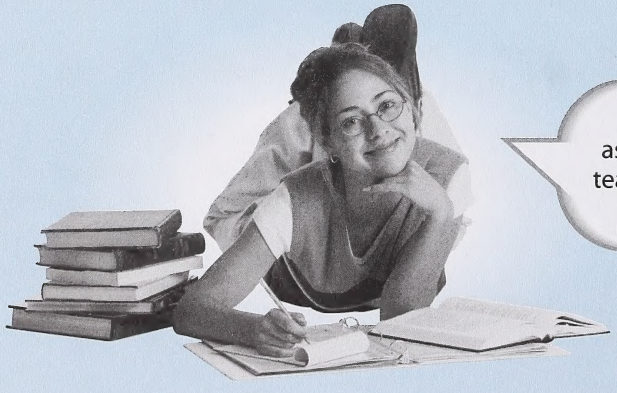


ASSESSMENT AND FEEDBACK

The Mathematics 14 course is carefully designed to give you many opportunities to discover how well you are doing. In every lesson you will be asked to turn to the Appendix to check your answers. Completing the lessons and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.

If you are having difficulty with a lesson, refer to the Answer Key in the Appendix for hints or help. As well as giving suggested answers to the questions, the Answer Key gives you more information about the questions.

Twice in each module you will be asked to give your teacher your completed assignments to mark. Your teacher will give you feedback on how you are doing.



After your teacher marks an assignment, be sure to review your teacher's comments and correct any errors you made.

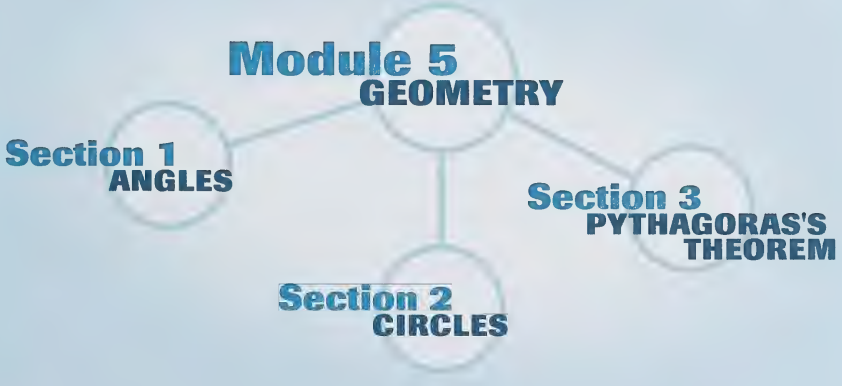
There will be a Final Test at the end of the course. You can prepare for the Final Test by completing the Review at the end of each module.

MODULE OVERVIEW

What is the shortest air route from Vancouver to London? If you look at most maps, your first reaction would be to say to fly east toward Newfoundland and then across the Atlantic Ocean to Britain. In fact, if you take a string and stretch it between Vancouver and London on a globe, you will see that the shortest route is along an arc of a circle that takes you over Nunavut and Greenland.

Charting a flight path between major cities or just navigating a short trip between communities in the North involves careful reading of maps and charts, using instruments to determine direction, and the application of mathematics in a variety of tasks.

In this module you will explore the geometry of angles, the relationships among angles, and how angles are used in navigation. As well, you will examine the geometry of the circle to find circumference and area. Lastly, you will investigate one of the most important geometrical relationships—Pythagoras's Theorem.



Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 5A

Section 1 Assignment	25 marks
Section 2 Assignment	25 marks

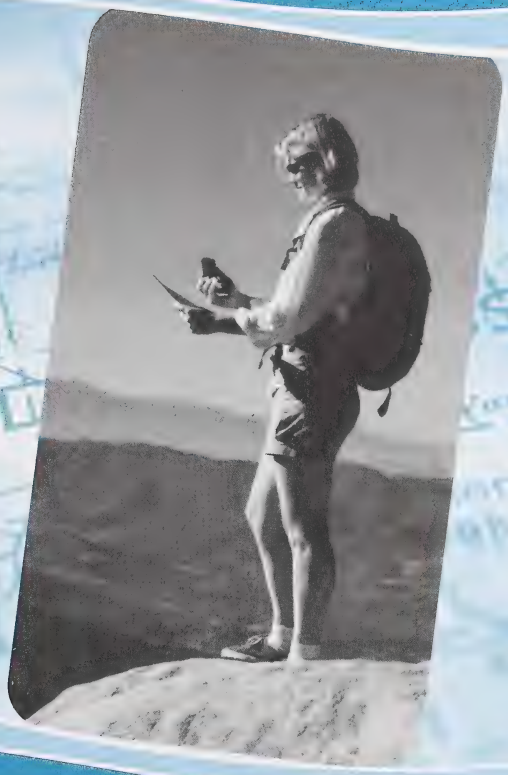
Assignment Booklet 5B

Section 3 Assignment	20 marks
Final Module Assignment	30 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 5A to your teacher before you begin Section 3. You will submit Assignment Booklet 5B to your teacher at the end of this module.

SECTION 1



Angles

Have you ever done any orienteering? Orienteering involves using maps and a compass to navigate between two points in unfamiliar country.

Did you know that a magnetic compass in most parts of Canada does not point due north, but rather points to the North Magnetic Pole? For example, near St. John's, Newfoundland and Labrador, the compass needle points at an angle of 23° towards the west, away from due north. Near Dawson, Yukon Territory, it points at an angle of 27° towards the east. Travellers in the wilderness have to be able to add or subtract angles to find their way!

To find your way in this module, you have to know the angles as well! You will construct and measure angles. You will explore how angles are classified and the relationships among them. Also, you will investigate how angles are used in navigation.

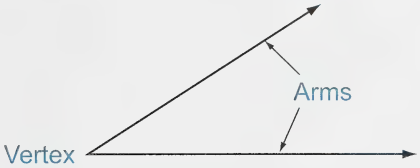
LESSON 1

Constructing and Measuring Angles

Today you will construct, measure, and classify angles.

Eileen is a doctor in Inuvik, Northwest Territories. As part of her duties, she is planning to charter an aircraft to fly to the communities at Holman and Sachs Harbour and then return to Inuvik. Look at her route on the map of the Northwest Territories above. Her planned flight forms a triangle. What are the approximate measures of the three angles in this triangle? How would you classify these angles based on these measures?

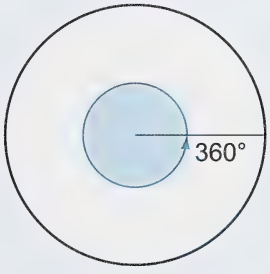
Remember, an **angle** is formed by two rays called **sides** or **arms** of the angle. The rays have a common endpoint called the **vertex** of the angle.



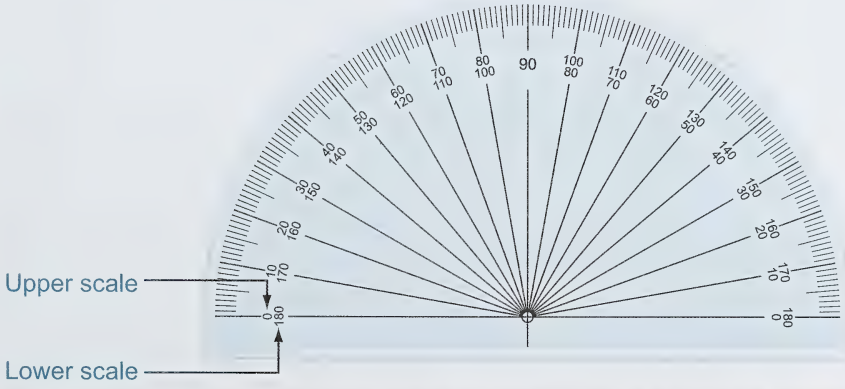
Looking Back

In previous mathematics courses, you measured angles using a **protractor**.

You will recall that there are 360° in a circle or in one complete rotation. So, in a semicircle or one half of a rotation, there are $0.5 \times 360° = 180°$.

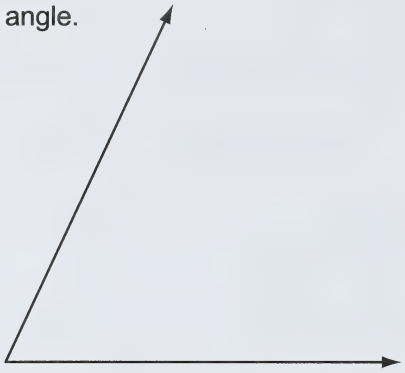


Most protractors are semicircular with two scales marked in **degrees** from 0° to 180°. Whether you use the upper scale or the lower scale depends on the orientation of the angle.

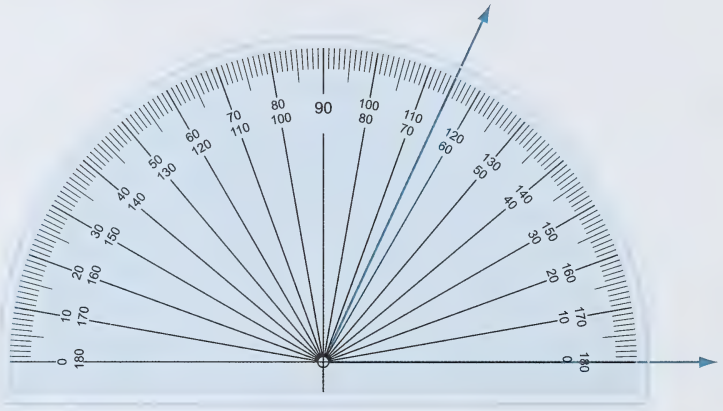


Example

Measure the following angle.



Begin by centring the protractor on the vertex of the angle and placing the base of the protractor along one arm of the angle.



To decide whether to read from the upper or lower scale, you can do either of the following:

- Estimate the size of the angle. This angle is less than 90°. The upper scale would give you a reading larger than 90°; so, use the lower scale.
- Start on the scale that has zero on the chosen arm of the angle. Use that scale for reading the angle size. So, use the lower scale.

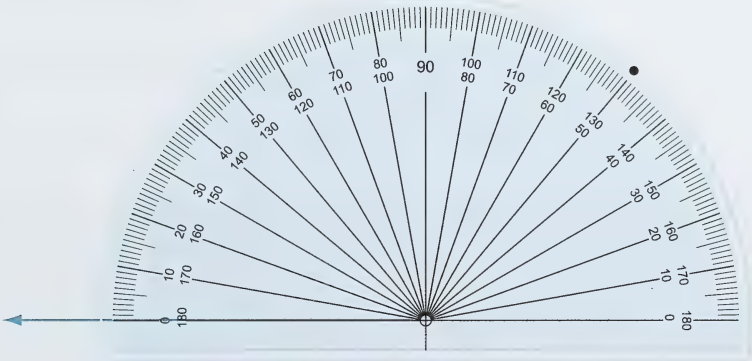
Do you agree that this is a 65° angle?

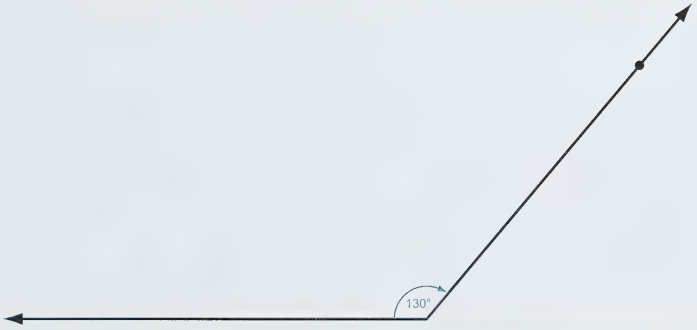
You can also use your protractor to draw angles.

Example

Draw a 130° angle using the given ray as the lower arm of the angle.

Begin by centring the protractor on the endpoint of the ray. Place the base of the protractor along the ray. This time, you will use the upper scale because it begins at 0° on the ray. Move clockwise along the upper scale until you reach 130°. Mark a dot at 130°. Then draw the second arm of the angle.

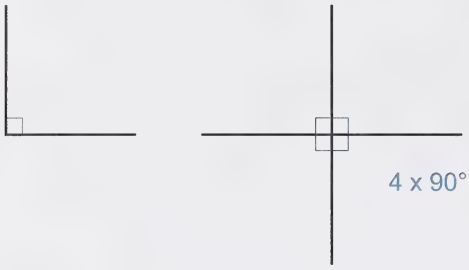




1. Turn to page 296 in your textbook. Do questions 1 and 2 of "Investigation." Before beginning "Investigation," you will need to make a square piece of paper to use. If you have a standard sized sheet of paper, just cut 2.5 inches from it. You will get a square piece of paper that measures 8.5 inches on a side. Otherwise, measure the width of the paper and cut off enough of the length to make a square page.

Do you remember the names for the different sizes of angles?

A **right angle** is one quarter of a complete rotation. It is 90° in measure. In the second diagram, there are four right angles.

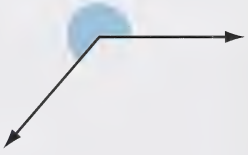


An **acute angle** is less than 90° in measure.

An **obtuse angle** is greater than 90° and less than 180°.

A **straight angle** is one half of a rotation. It is 180° in measure.

A **reflex angle** is greater than 180° and less than 360°.



A **full rotation** is 360°.

Next, you will practise using these terms.

Turn to page 298 in your textbook. Read Example 1.

2. Turn to page 301 in your textbook. Do questions 1, 2, 3.a., and 3.b. of "Put into Practice."

Check your answers on pages 55 to 57 in the Appendix.

In this lesson you constructed, measured, and classified angles. To review the concepts in this lesson, work through "Lesson 16: Angles—Construct and Measure" on the CD-ROM that accompanies your textbook.

Turn to the Section 1 Assignment in Assignment Booklet 5A. Answer questions 1 and 2.

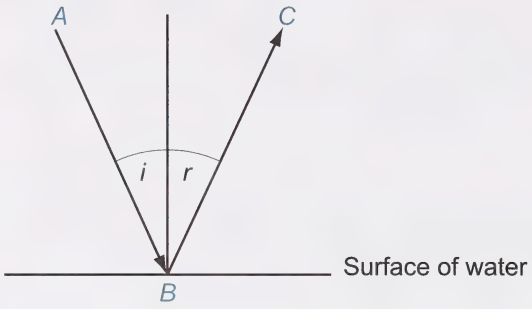
LESSON 2

Angle Relationships

Today you will investigate relationships among angles.

Early in the morning or just before sunset is often the best time to see reflections in the surface of water. Did you know that these reflections occur in part because of the relationships between angles? The angle at which light strikes the surface of the water is equal to the angle at which the light is reflected.

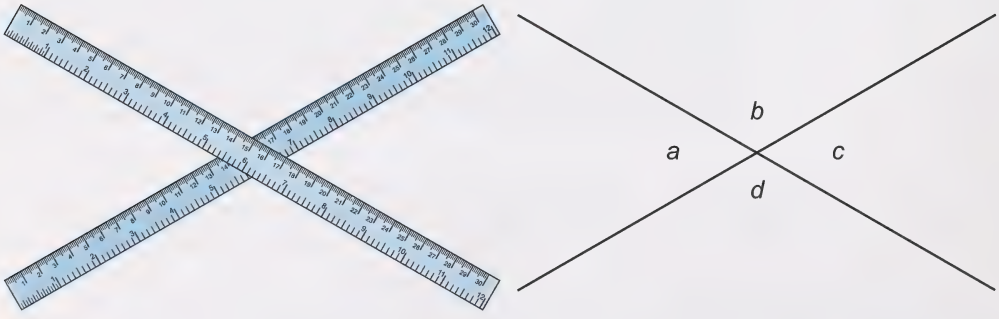
In the following diagram, the light ray bouncing off the surface of the water forms a pair of equal angles, *i* and *r*.



You can think of these angles as halves of the large angle *ABC*. The vertical line that divides angle *ABC* into halves is called an **angle bisector**.

Also, because angles *i* and *r* are on opposite sides of a common arm and share a common vertex, these angles are an example of **adjacent angles**. Adjacent angles do not have to be equal in measure. In the following diagram, angles *a* and *b* are adjacent angles but are different sizes. In this lesson you will work with adjacent angles.

You will also be working with angles that are on opposite sides of intersecting lines. If you were to lay one ruler across another ruler, you would form two pairs of **opposite angles**.



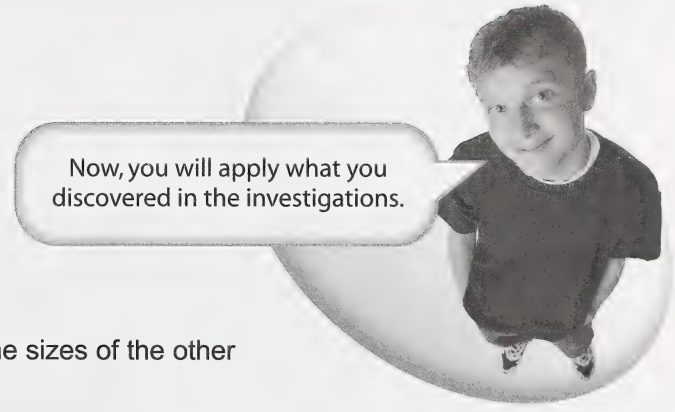
In the diagram, angles *a* and *c* are opposite angles, and angles *b* and *d* are opposite angles.

How do the sizes of opposite angles compare? To find out, complete the following investigations.

1. Turn to pages 302 and 303 in your textbook. Do questions 1 to 3 of "Investigation."

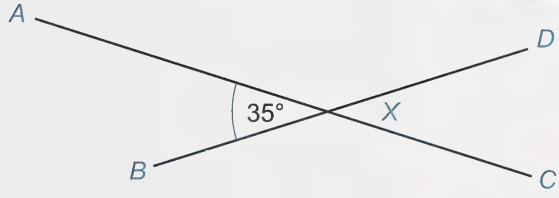
2. Turn to page 304 in your textbook. Do questions 1 to 3 of "Investigation."

Check your answers on pages 58 and 59 in the Appendix.



Example

If $\angle AXB = 35°$, calculate the sizes of the other three angles.



$\angle BXA$ and $\angle AXD$ are adjacent angles. They form the straight angle BXD.

So, $\angle BXA + \angle AXD = 180°$.

$$35° + \angle AXD = 180°$$
$$\angle AXD = 180° - 35°$$
$$= 145°$$

$\angle AXB$ and $\angle CXD$ are opposite angles. These angles are equal in size.

$$\angle CXD = 35°$$

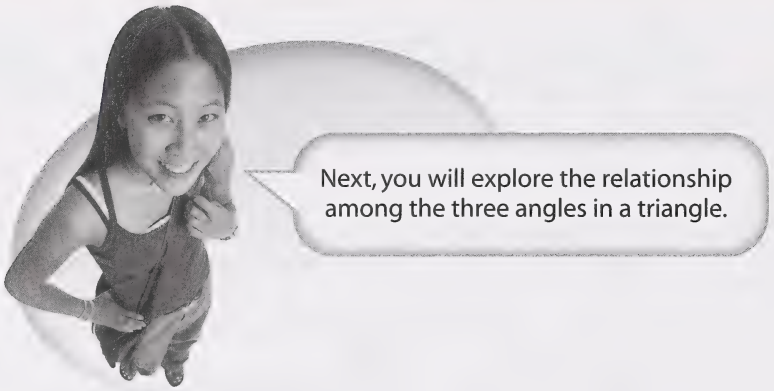
$\angle AXD$ and $\angle BXC$ are opposite angles. These angles are equal in size.

$$\angle BXC = 145°$$

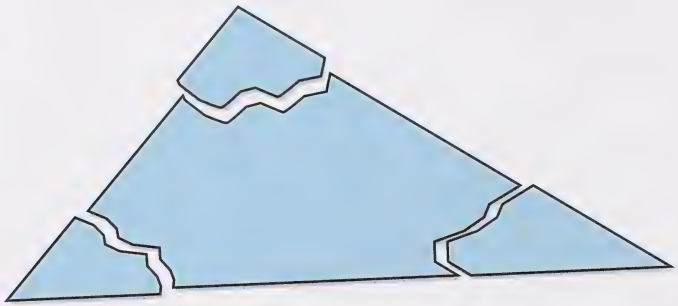
Turn to page 305 in your textbook. Work through Example 1.

3. Turn to page 306 in your textbook. Do questions 1 to 3 of "Put into Practice."

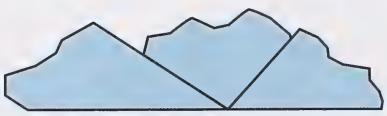
Check your answers on pages 59 and 60 in the Appendix.



Draw any large triangle on an 8.5" × 11" sheet of paper. Use scissors to cut it out.



Tear off each corner, and then fit the corners together in any order.



Do the corners fit together to form a straight angle? Repeat the steps with a different shape of triangle—perhaps one that contains an obtuse angle. What did you discover?

You have discovered that the sum of the three interior angles of a triangle is 180°. You can use this angle relationship to solve a variety of problems.

4. Turn to page 305 in your textbook. Do question 1 of "Investigation."

Read the summary in the yellow box on page 305 and work through Example 2.

5. Turn to page 307 in your textbook. Do questions 4 and 5 of "Put into Practice."

Check your answers on pages 60 to 62 in the Appendix.

In this lesson you explored angle relationships. To review the concepts from this lesson, work through "Lesson 17: Angles—Reasoning" on the CD-ROM that accompanies your textbook.

Turn to the Section 1 Assignment in Assignment Booklet 5A. Answer questions 3, 4, 5, and 6.

LESSON 3

Angles in Navigation

Today you will investigate how angles are used in air navigation.

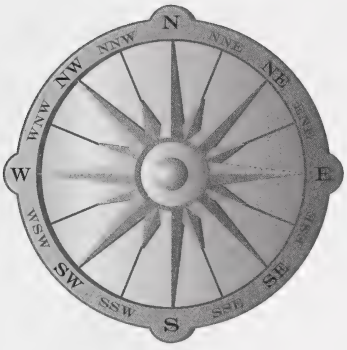
To many aviation buffs, the Avro Arrow, which was designed and built in Canada in the 1950s, was one of Canada's premier military aircraft. Many argue that it was the fastest and most advanced supersonic jet of its time.

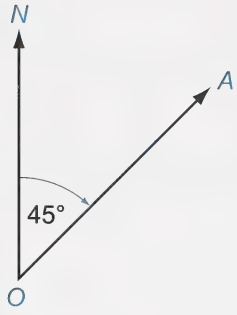
If you have access to the Internet, you can find out more about Canada's aviation history at the Canadian National Aviation Museum website:

http://www.nmstc.ca/nam/

Although you may have never been on a military aircraft, you probably have or will travel by commercial airliner. Did you know that to determine flight paths to your travel destinations, pilots and onboard computer navigation systems use angles called **bearings**? A bearing is the angle of the course taken, measured clockwise from due north.

Study the following diagram to discover the relationship between compass directions and bearings.





The bearing of an aircraft flying along OA is 45°. If you look at the compass on the left, you can see that the aircraft is headed NE.

Example

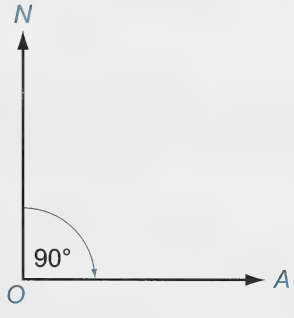
In what direction is an aircraft headed if its bearing is

a. 90° **b.** 180°
c. 315° **d.** 0°

Solutions

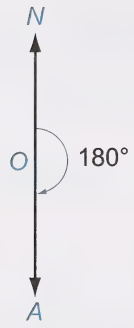
In each diagram, the aircraft is headed from O to A.

a.

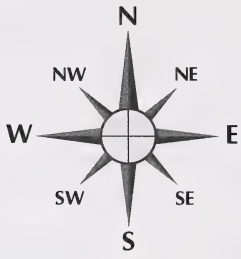


A bearing of 90° is due east.

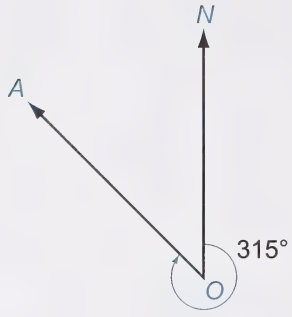
b.



A bearing of 180° is due south.



c.



A bearing of 315° is NW.

d.

A bearing of 0° is due north.

Now try your hand at translating bearings into compass directions.

1. Convert each bearing into a compass direction.

a. 135°
b. 270°
c. 225°
d. 22.5°

Check your answers on page 62 in the Appendix.

Because a bearing can be any angle between 0° and 360°, in most instances it is easier to use than a compass direction.

Turn to page 308 in your textbook. Work through Example 1.

2. Turn to pages 308 and 309 in your textbook. Do questions 1 to 3 of "Put into Practice."

Check your answers on page 63 in the Appendix.

3. Turn to pages 310 and 311 in your textbook. Work through Examples 2 and 3, and then do questions 4.a., 4.d., 5, and 6 of "Put into Practice."

Check your answers on pages 63 to 65 in the Appendix.

In this lesson you explored how angles are used in specifying bearings.

Turn to the Section 1 Assignment in Assignment Booklet 5A. Answer question 7.

CONCLUSION

In this section you explored angles and their relationships. You estimated, measured, and drew angles using a protractor. You classified angles by size. You also investigated and determined the measures of angles in a diagram. These diagrams included adjacent angles, opposite angles, and angles in triangles.

Finding your way in the wilderness with a map is a lot like following mathematical diagrams. You measure distances and decide where to turn and how much to turn in order to get to your destination.

The two hikers in the photo above have a map and a trail to follow. They will likely get to their destination without trouble. Like good orienteers, they came prepared with the needed equipment to get where they are going safely.

SECTION 2

Circles

Astronauts who walked on the moon were struck by how beautiful Earth looked as it rose over the lunar horizon. When seen from the distances of space, Earth appears as a perfect circle. Irregularities in its surface vanished, and astronauts realized that the people of the world share a small planet.

Although the astronauts were the first people to see Earth as a circle in the sky, ancient peoples also knew Earth was round. They saw the curved shadow of Earth during lunar eclipses. Early Greek mathematicians even calculated the radius of the Earth.

People often think that these same mathematicians developed geometry, but really most of the work was done much earlier. Much of the development of geometry also came from everyday living. It came from answers to questions like "How do you make rooms square?" or "How do you find your farm after it has been flooded by the Nile River?" Modern society builds on these thousands of years of experience.

In this section you will investigate circles. You will explore the relationships among radius, diameter, circumference, and area. You will apply these relationships in a variety of problem situations.

LESSON 1

Circumference of Circles

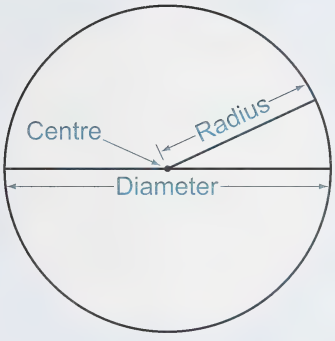
Today you will explore the relationship among the radius, diameter, and circumference of a circle.

Jeanne has just purchased an odometer for her mountain bike. As part of the installation, she mounted a pin on one of the spokes on her front wheel. Every time the wheel turns, the pin strikes a cog on the odometer. The cog advances and the odometer translates this into the distance the bike travels when the wheels rotate once. The odometer is set for a wheel diameter of 26 inches.

The distance travelled by the bike during one rotation of the front wheel is the **circumference** of the wheel. The circumference of the wheel can be determined from its diameter. Before you calculate this distance, you will need to review some of the terminology and concepts associated with circles.

Looking Back

A **circle** is a set of all points that are the same distance from a fixed point, which is called the **centre of the circle**. The distance each point lies from the centre is the **radius** of the circle. The **diameter** of the circle (the distance across the circle) is twice the radius.



A carpenter uses this definition to draw circles or arcs on sheets of plywood. The carpenter simply drives a small nail into the plywood. This will be the centre of the circle. Then, the carpenter attaches a pencil to the nail with a string. The distance the pencil lies from the centre is the radius of the circle. The carpenter simply pulls the string tight and uses the pencil to trace out the circle.

The following example illustrates the relationship between the radius and the diameter of a circle.

Example

A square box is designed to hold a cylindrical vase. If the opening at the top of the box is 10 cm on each side, what is the maximum diameter and radius of a vase that will fit in the box?

The maximum diameter of the vase is the width of the square opening, or 10 cm. Therefore, the radius of the vase is $10 \text{ cm} \div 2 = 5 \text{ cm}$.

Turn to page 312 in your textbook. Read the summary in the box at the top of the page.

1. Do questions 1 to 3 of "Put into Practice" on page 312.

Check your answers on page 66 in the Appendix.

How would you estimate the circumference of a circle from its radius?

2. Turn to page 313 in your textbook. Do questions 1 to 3 of "Investigation."

Check your answers on pages 66 and 67 in the Appendix.

3. Turn to page 314 in your textbook. Do questions a. to e. of "Investigation."

Check your answers on page 67 in the Appendix.

Turn to page 314 in your textbook and read the summary in the blue rectangle to review the exact relationship between circumference and diameter.

Through your investigations you discovered that, for any circle, the relationship between the circumference (C) and the diameter (d) is $\frac{C}{d} = \pi$, where $\pi = 3.141\,592\,653\ldots$. This relationship can be rewritten to solve for either the circumference or the diameter.

To find the circumference of a circle for which you know the diameter, you will have to solve this relationship for C.

$$\frac{C}{d} = \pi$$

$$\frac{C}{\cancel{d}_1} \times \cancel{d}^1 = \pi \times d$$

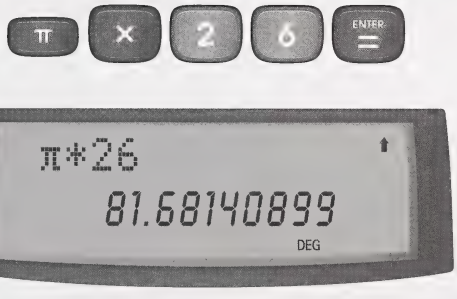
$$C = \pi d$$

Example

Determine the circumference, correct to the nearest inch, of a bike wheel with a 26-inch diameter.

$$C = \pi d$$
$$= \pi \times 26$$

Use your calculator to evaluate the circumference. Press these keys.



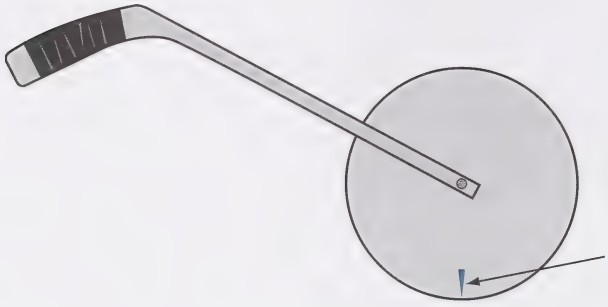
$C \doteq 81.681\,408\,99$

The circumference of the wheel is about 82 inches.

Example

Rod is on the planning committee for a community sports day. To help measure out distances for various events, Rod has decided to make a trundle wheel that will measure out 1 m every time it rotates. For the handle, he plans to use an old hockey stick. He will bolt the stick to a plywood disk.

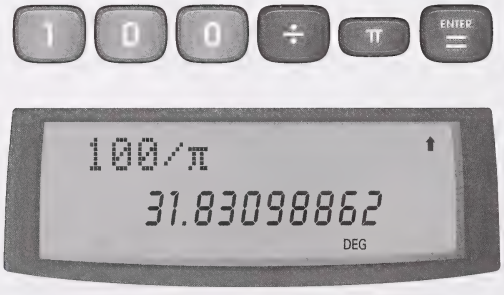
What is the diameter of the disk that Rod will have to cut out? Round your answer to the nearest tenth of a centimetre.



The circumference of the wheel is 1 m or 100 cm.

$$C = \pi d$$
$$100 = \pi \times d$$
$$d = \frac{100}{\pi}$$

Use your calculator. Press these keys.



$d \doteq 31.830\,988\,62$

The diameter of the wheel is about 31.8 cm.

Turn to page 315 in your textbook. Work through Examples 1 and 2.

4. Turn to pages 316 to 318 in your textbook. Do questions 4 to 8 and question 10 of "Put into Practice."

Check your answers on pages 67 to 72 in the Appendix.

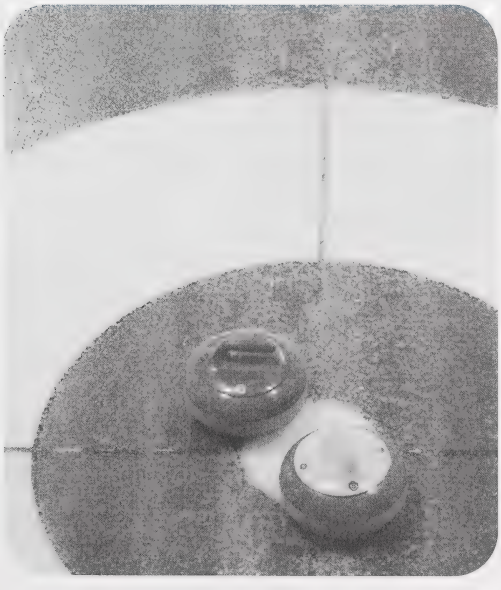
In this lesson you explored circles and their circumference. To review the concepts from this lesson, work through "Lesson 18: Circles—Circumference" on the CD-ROM that accompanies your textbook.

Turn to the Section 2 Assignment in Assignment Booklet 5A. Answer questions 1, 2, and 3.

LESSON 2

Area of a Circle

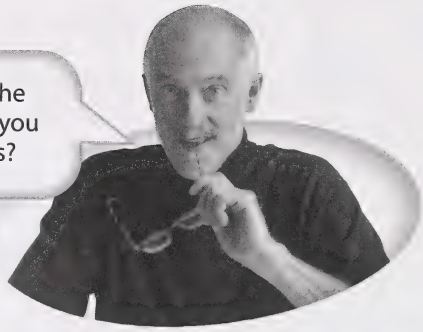
Today you will investigate the area of a circle.



Beverly is helping her father make the ice at the local curling rink. They are preparing to paint the circles at each end of a sheet of ice. The button and the 8-foot circles are white. The 4-foot circles are red and the 12-foot circles are blue.

Beverly thinks they will need three times as much blue paint as red paint because the diameter of the 12-foot circle is three times as large as the diameter of the 4-foot circle. Is she right?

To answer Beverly's question, you need to know the formula for the area of a circle. But first, how could you estimate the area of a circle if you knew its radius?



If you have access to the Internet, you can find out more about the sport of curling at the following website:

http://www.curling.ca

1. Turn to page 319 in your textbook. Do questions 1 and 2 of "Investigation."

Check your answers on page 72 in the Appendix.

In the investigation you found out that you can estimate the area of a circle by multiplying the square of its radius by 3.

Example

What is the approximate area of the "house" in the sport of curling? (**Note:** The "house" is the series of coloured rings at either end of a sheet of curling ice.)

The diameter of the house is 12 feet. Therefore, the radius, r, is 6 feet.

The square of the radius is $6 \text{ ft} \times 6 \text{ ft} = 36 \text{ ft}^2$.

An estimate of the area is $3r^2 = 3 \times 36 \text{ ft}^2$

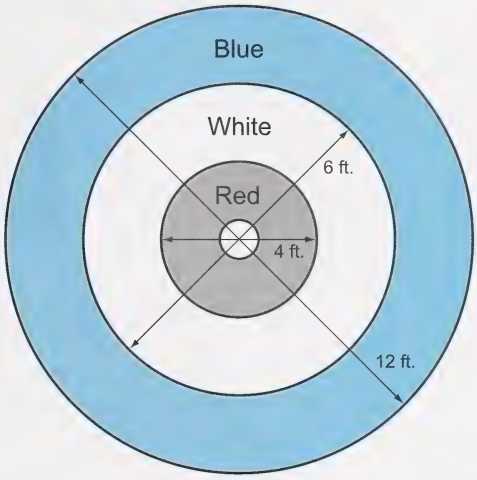
$= 108 \text{ ft}^2$

The house is about 108 ft^2 in area.

To find the exact area (A) of a circle using the radius (r), you must use the formula $A = \pi r^2$, which is just a little more than the estimate $A \simeq 3r^2$.

Example

For the sport of curling, determine how many times larger in area the blue 12-foot ring is than the red 4-foot ring.

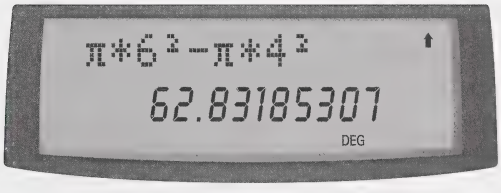


Twelve feet and 4 feet refer to the outside diameters of these rings. The corresponding radii are 6 feet and 2 feet.

First find the area of the 12-foot ring.

$$\text{area of 12-foot ring} = \text{area of house} - \text{area of 8-foot ring}$$
$$= \left(\pi \times 6^2\right) - \left(\pi \times 4^2\right)$$

Use your calculator. Press these keys.

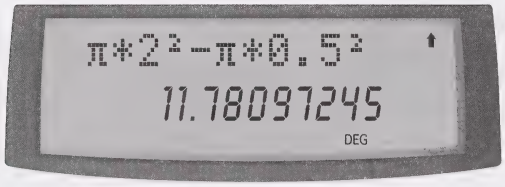


The area of the 12-foot ring is about $62.832\ \text{ft}^2$.

Next, find the area of the 4-foot ring.

$$\text{area of 4-foot ring} = \text{area of 4-foot ring} - \text{area of button}$$
$$= \left(\pi \times 2^2\right) - \left(\pi \times 0.5^2\right) \quad \leftarrow \text{Radius of button} = 0.5\ \text{ft}$$

Use your calculator. Press these keys.



The area of the 4-foot ring is about 11.781 ft^2.

Divide the area of the 12-foot ring by the area of the 4-foot ring.

$$62.832 \div 11.781 = 5.33\overline{3}$$

The area of the 12-foot ring is about 5.333 times as large as the area of the 4-foot ring.

In the introduction to this lesson, Beverly was wrong. The 12-foot circles would require over 5 times as much paint as the 4-foot circles.

Turn to page 320 in your textbook. Work through Examples 1 and 2. Note that the correct answer in Example 2 is 6648 cm^2.

2. Turn to pages 321 and 322 in your textbook. Do questions 1 to 5 of "Put into Practice."

Check your answers on pages 72 to 74 in the Appendix.

You can also calculate the radius and diameter of a circle if you know its area.

Turn to page 323 in your textbook. Work through Example 3.

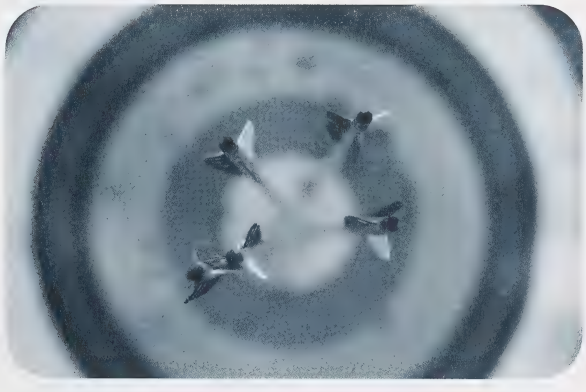
3. Turn to pages 324 to 326 in your textbook. Do questions 7, 8, 9, 11, 12, and 13 of "Put into Practice."

Check your answers on pages 74 to 78 in the Appendix.

In this lesson you explored circles and their area. To review the concepts from this lesson, work through "Lesson 19: Circles—Area" on the CD-ROM that accompanies your textbook.

Turn to the Section 2 Assignment in Assignment Booklet 5A. Answer questions 4, 5, and 6.

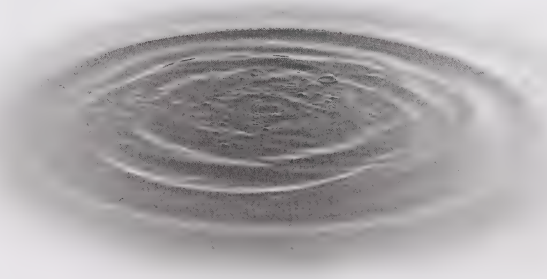
When you are finished, submit Assignment Booklet 5A to your teacher to be marked.



CONCLUSION

In this section you explored the circle. You investigated the relationships among radius, diameter, circumference, and area. For area and volume, you examined approximation techniques and the application of formulas. You also applied these formulas to solve a variety of problem situations.

The circle is an inseparable part of society: in gears, wheels, pulleys, and axles; from the design of industrial vessels to household dishes; in art and jewellery; and in pipes and pipelines, just to name a few. But, have you thought of the circle in nature? The circle appears in the stars, in the orbits of planets, and even in the spread of ripples in a pond. In the photograph above, the circles are the tracks left by stars in the night sky as Earth spins on its axis.



SECTION 3

Pythagoras's Theorem

When people are asked to think of mathematicians and scientists, the two names mentioned most often are Einstein and Pythagoras. As a matter of fact, most people can even recite the formulas for which they are remembered: $E = mc^2$ and $c^2 = a^2 + b^2$.

Did you know that Pythagoras lived over 2500 years ago? Geometry was just one of his many interests. He came up with a formal proof of the theorem that was named after him. But this relationship, involving the sides of a right triangle, was known and applied centuries before—in ancient Egypt to survey land along the Nile, and in ancient Babylon over 3800 years ago!

If you have access to the Internet, you can find out more about Pythagoras and his theorem at the following website:

http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html

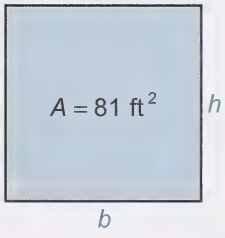
In this section you will explore Pythagoras's Theorem and its application in a variety of practical situations.

LESSON 1

Square Roots

Today you will investigate how to estimate square roots and how to obtain them using a calculator.

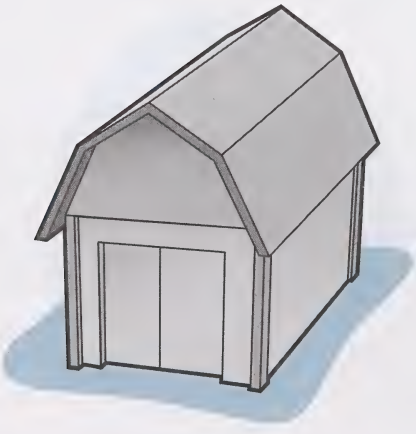
Brianna is helping her father build a garden shed in their backyard. Her parents want a square shed with a floor area of 81 ft^2. What will be the dimensions of the base of the shed?



Since $A = bh$, and for a square $b = h$,

$$81 = b^2$$

$$b = \sqrt{81}$$



Brianna will use her calculator to find the **square root** of 81. She knows that a square root of a number is the value that when squared gives you the original number. She presses the following sequence of keys on her calculator.

The base of the shed should be 9 ft by 9 ft. This answer is correct because $9 \times 9 = 81$.

When Brianna's father asked her what the dimensions of the square base would be if the area were 90 ft^2, she said it would be about 9.5 ft by 9.5 ft. She did not use her calculator to obtain this estimate. How did she do it?

Example

Estimate $\sqrt{90}$.

You know that $\sqrt{90}$ lies between 81 and 100. These numbers are **perfect squares**, numbers that have whole number square roots.

$81 < 90 < 100$

$9^2 < 90 < 10^2$

So, $9 < \sqrt{90} < 10$.

A reasonable estimate is 9.5 since $9.5^2 = 90.25$.

1. Turn to pages 351 and 352 in your textbook. Work through the example on page 351 and then do questions 1 to 3 of "Put into Practice."

2. Turn to page 353 in your textbook. Read the explanation about estimation in the blue box and then do question 4 of "Put into Practice."

Check your answers on pages 78 and 79 in the Appendix.

Looking Back

In many questions involving square roots, you will need to round your answers.

Example

Round each of the following numbers to three decimal places.

a. 3.4568 **b.** 3.4565 **c.** 3.4564

Solutions

a. Look at the fourth decimal place.

3.4568 rounds to 3.457 since the fourth decimal place is 8, a number that is greater than or equal to 5.

b. Look at the fourth decimal place.

3.4565 rounds to 3.457 since the fourth decimal place is 5.

c. Look at the fourth decimal place.

3.4564 rounds to 3.456 since the fourth decimal place is 4, a number that is less than 5.

You will use your rounding skills to answer the following questions.

3. Turn to page 353 in your textbook. Do questions 5 and 6 of "Put into Practice."

Check your answers on page 79 in the Appendix.

In this lesson you determined square roots. This is a important skill in applying Pythagoras's Theorem.

Turn to the Section 3 Assignment in Assignment Booklet 5B. Answer question 1.

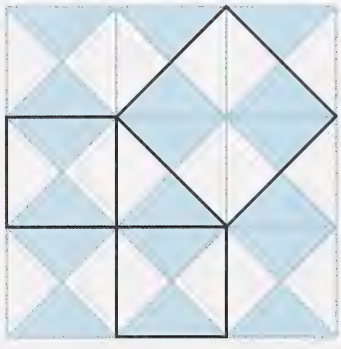
LESSON 2

Pythagoras's Theorem

Today you will explore the meaning of Pythagoras's Theorem.

Chandra was installing tiles in her bathroom. She noticed that Pythagoras's Theorem could be seen in the pattern she was making. To show this to her younger brother, she outlined a right triangle using masking tape. Then, she outlined the squares that were formed using each of the three sides of the triangle.

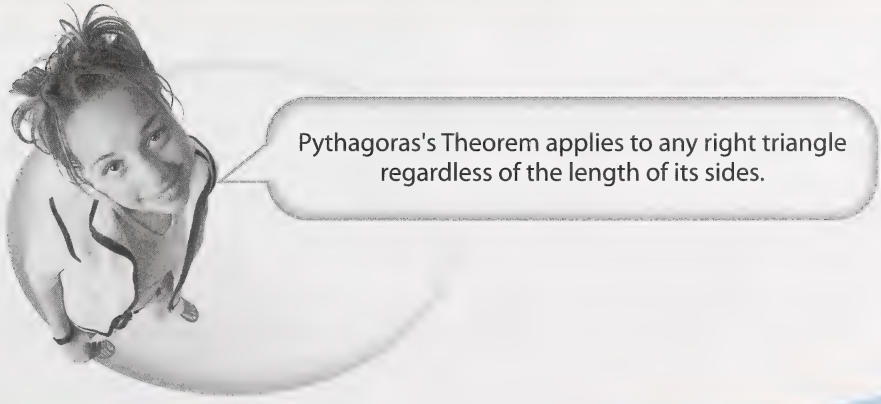
She asked her brother to count the small white and blue triangles in the largest of the three squares. Her brother knew that the largest square was the square on the **hypotenuse**, the longest side of the right triangle. He counted eight small triangles.



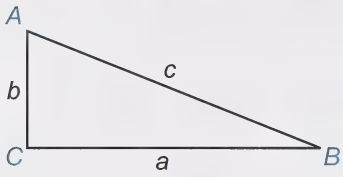
Then, she asked her brother to count the total number of small white and blue triangles in the two squares on the other two sides of the right triangle. He counted four triangles in each of these squares—eight altogether, the same number that were in the square on the hypotenuse!

Pythagoras's Theorem

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.



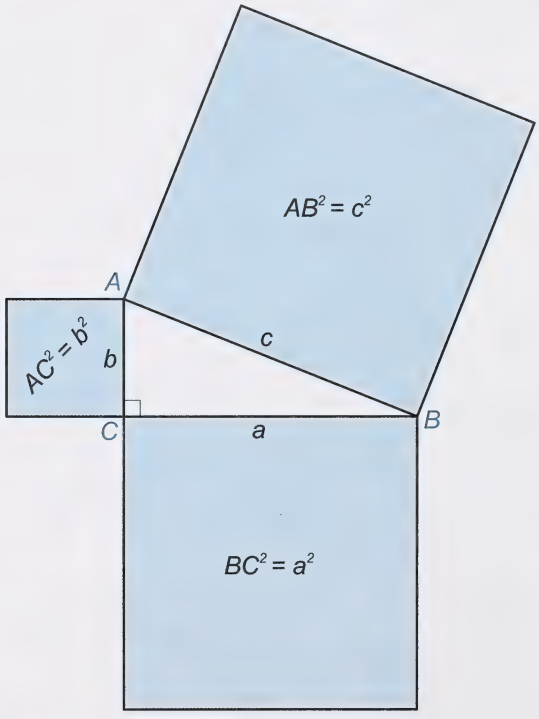
For example, look at the following right triangle.



In this triangle, the hypotenuse is *AB*. You can represent the length of this hypotenuse by *AB* or *c*. The small letter *c* may be used because the hypotenuse lies opposite angle *C*.

The other sides are *AC* and *BC*. You can use the letter *b* for side *AC* because this side lies opposite angle *A*. You can use the letter *a* for side *BC* because this side lies opposite angle *A*.

Now, construct squares on each side of the triangle.



So, for ΔABC,

$$AB^2 = AC^2 + BC^2$$

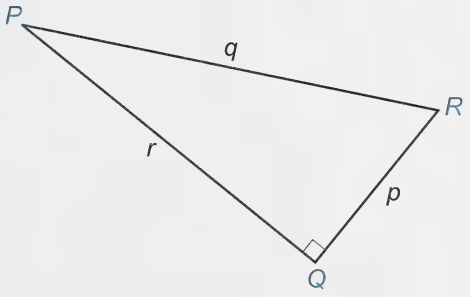
or

$$c^2 = a^2 + b^2$$

If you change the letters, you will change the statement of the theorem.

Example

What is the statement of Pythagoras's Theorem for ΔPQR?

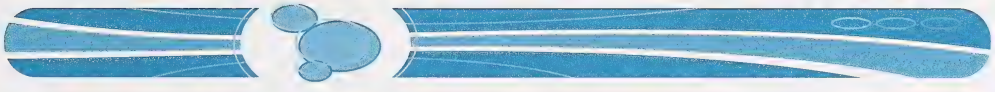


The hypotenuse is PR or q. The other sides are QR or p and PQ or r.

Therefore, $PR^2 = QR^2 + PQ^2$

or

$$q^2 = p^2 + r^2$$



Turn to page 354 in your textbook. Read this page and then do questions 1 and 2 of "Investigation."

Check your answers on page 80 in the Appendix.

In this lesson you explored Pythagoras's Theorem. In the next lesson you will use Pythagoras's Theorem to solve problems.

Turn to the Section 3 Assignment in Assignment Booklet 5B. Answer question 2.

LESSON 3

Applying Pythagoras's Theorem

Today you will apply Pythagoras's Theorem in a variety of problem situations.

Pythagoras's Theorem has been used to solve problems for at least 4000 years. Ancient cuneiform tablets found in present-day Iraq and Iran indicate that Babylonian mathematicians had a sophisticated knowledge of square roots and Pythagorean relationships.

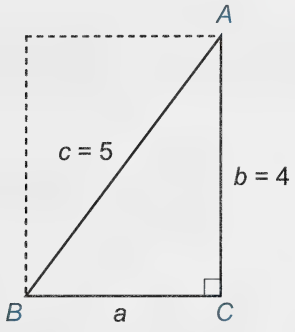
Cuneiform is the first known form of writing. As the photo shows, it was a series of impressions in a clay tablet. You can find out more at the following website:

http://www.upenn.edu/museum/Games/cuneiform.html

The following example shows the modern solution for one of the simpler problems from a cuneiform tablet that is in The British Museum in London, England.

Example

The diagonal of a rectangle is 5 units. If the length is 4 units, what is the width?



$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
5^2 &= a^2 + 4^2 \\
25 &= a^2 + 16 \\
25 - 16 &= a^2 + 16 - 16 \\
9 &= a^2 \\
a^2 &= 9 \\
a &= \sqrt{9} \\
a &= 3
\end{aligned}
$$

Therefore, the width is 3 units.

As you saw in the previous example, the whole numbers 3, 4, and 5 are the lengths of the sides of a right triangle. These whole numbers satisfy Pythagoras's Theorem. Recall, $3^2 + 4^2 = 5^2$. These three numbers are just one example of a **Pythagorean triple**. In fact, there are an infinite number of such triples.

1. Show that each of the following is a Pythagorean triple. Draw the corresponding right triangle for each triple.

a. 45, 60, 75
b. 5, 12, 13
c. 8, 15, 17

Check your answers on pages 80 and 81 in the Appendix.

If you have access to the Internet, you can find out more about Pythagoras's Theorem in Babylonian mathematics at the following website:

http://www-gap.dcs.st-and.ac.uk/~history/HistTopics/Babylonian_Pythagoras.html

Turn to page 356 in your textbook. Work through Examples 1 and 2.

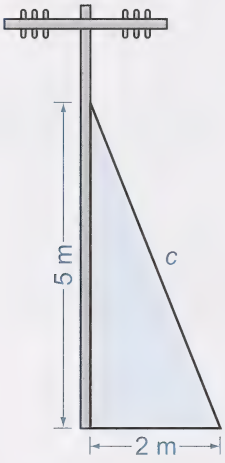
2. Turn to pages 355 to 358 in your textbook. Do questions 1 to 6 of "Put into Practice."

Check your answers on pages 81 to 83 in the Appendix.

Not all right-triangle problems involve whole numbers.

Example

A support wire for a power pole is connected to the pole at a point 5 m above the ground. The other end of the wire is staked to the ground 2 m from the foot of the pole. What is the length of the wire? Round your answer to the nearest tenth.



$$c^2 = 5^2 + 2^2$$
$$c^2 = 25 + 4$$
$$c^2 = 29$$
$$c = \sqrt{29}$$
$$c \doteq 5.385\,164\,807$$

The length of the wire is approximately 5.4 m.

3. Turn to pages 358 to 360 in your textbook. Do questions 7 to 13 of "Put into Practice."

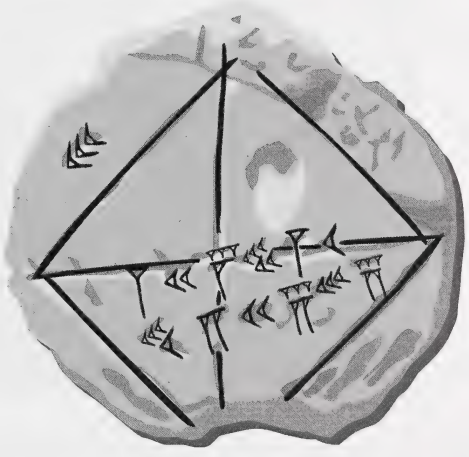
Check your answers on pages 83 to 86 in the Appendix.

In this lesson you applied Pythagoras's Theorem. To review the concepts from this lesson, work through "Lesson 21: Pythagoras" on the CD-ROM that accompanies your textbook.

Turn to the Section 3 Assignment in Assignment Booklet 5B. Answer questions 3, 4, 5, and 6.

CONCLUSION

In this section you explored square roots and Pythagoras's Theorem. You examined the geometric interpretation of this theorem. Also, you applied this theorem to solve a variety of everyday problems.



The calculation of square roots and solutions to problems based on the Pythagorean relationship date back at least 4000 years to ancient Babylon. The solution illustrated on the cuneiform tablet above is for finding the length of the diagonal of a square measuring 30 units on each side. The answer is given using base-60 Babylonian symbols and is correct to 5 decimal places, and it was determined without a calculator! With the skills you acquired in this section and your calculator, you can also solve this problem.

MODULE SUMMARY

The theme of this module was geometry. In Section 1 you explored angles, their classification, their relationships, and their application in a variety of problems. These problems involved adjacent and opposite angles, angles in triangles, and angles used in navigation.

In Section 2 you investigated the circle, its properties, and its circumference and area.

In Section 3 you estimated and evaluated square roots. You used these skills to explore Pythagoras's Theorem and how it is used to solve everyday problems.

The geometry covered in this module is used every day in all sorts of situations. Who would have thought that you'd need to know about π to get a curling rink ready for use or to design a package to store tennis balls? Would you have known about using angles to name airport runways?

As the flight returns from London to Vancouver over northern Canada, it's good to know that everyone agrees on how to name the runway the plane will land on. It's another of the hidden mathematical ties to our modern world.

Turn to Assignment Booklet 5B and complete the Final Module Assignment.

When you are done, submit Assignment Booklet 5B to your teacher to be marked.

REVIEW

This Review will help you apply what you learned in Module 5 and prepare for the Final Test. Read over the skills checklist for this module. Use this list to guide your study and to help you decide how much of the Review you should complete.

Skills Checklist

Angles

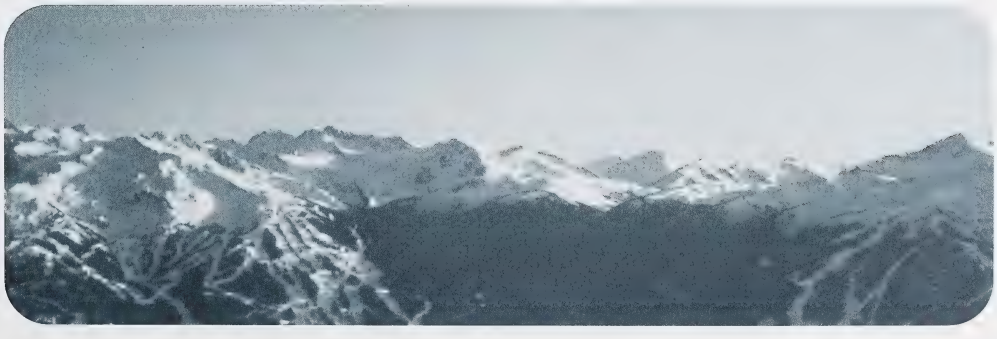
- ❑ Estimate, measure, and draw angles using a protractor.
- ❑ Determine the measures of angles in a diagram.

Circles

- ❑ Measure the diameters, radii, and circumferences of circles and establish the relationships among them.
- ❑ Calculate the areas of a circle.
- ❑ Solve problems involving the radii, diameters, areas, and circumferences of circles.

Pythagoras's Theorem

- ❑ Use concrete materials and diagrams to verify the Pythagorean relationship.
- ❑ Use the Pythagorean relationship to calculate the measure of the third side of a right triangle, given the other two sides, in 2-D applications.



If you need additional work to master the material in this module, work through the following lessons on the CD-ROM that accompanies your textbook:

- "Lesson 16: Angles—Construct and Measure"
- "Lesson 17: Angles—Reasoning"
- "Lesson 18: Circles—Circumference"
- "Lesson 19: Circles—Area"
- "Lesson 21: Pythagoras"

1. Turn to pages 327 to 329 in your textbook. Do questions 1 to 9 of "Review."

2. Turn to pages 362 and 363 of your textbook. Do questions 8 to 12 of "Review."

Check your answers on pages 86 to 93 in the Appendix.

MATHEMATICS

14

Appendix

GLOSSARY

ANSWER KEY

IMAGE CREDITS

Glossary

acute angle: an angle that is less than 90° in measure

adjacent angles: angles that are on opposite sides of a common arm

angle: a geometric figure formed by two rays with a common endpoint

angle bisector: a line that divides an angle into halves

arm of an angle: one of the two rays forming an angle

bearings: navigational angles that are measured clockwise from due north

centre of a circle: the common endpoint of the radii of a circle

circle: the set of all points that are the same distance from a fixed point

circumference: the distance around a circle

degree: unit of angle measure

diameter: the distance across a circle

full rotation: an angle that is 360° in measure

hypotenuse: the longest side of a right triangle

obtuse angle: an angle that is between 90° and 180° in measure

opposite angles: angles that are on opposite sides of two intersecting lines

perfect square: a number that has a whole number square root

protractor: a device for measuring angles

Pythagoras's Theorem: the theorem that describes the relationship among the sides of a right triangle

Pythagorean triple: three whole numbers that satisfy Pythagoras's Theorem

radii: plural of radius

radius: the distance each point on a circle lies from the centre of the circle

reflex angle: an angle that is between 180° and 360° in measure

right angle: an angle that is 90° in measure

side of an angle: an arm of an angle

square root of a number: the value that when squared gives you the original number

straight angle: an angle that is 180° in measure

vertex of an angle: the common endpoint of the two rays forming an angle

Answer Key

Section 1: Angles

Lesson 1: Constructing and Measuring Angles

1. Textbook, page 296, "Investigation," questions 1 and 2

1. a. $\angle NMS = 180°$

b. $\angle NMW = \angle NME = \angle SMW = \angle SME = 90°$

2. a. $\angle NMA = 45°$

b. $\angle EMA = 45°$ (So are $\angle SMB$ and $\angle BMW$.)

c. $\angle AMS = 45° + 90°$
$= 135°$

d. $\angle WMA = 135°$ (So are $\angle BME$, $\angle WMA$, $\angle BMN$, and $\angle SMA$.)

e. $\angle NMA = 360° - 45°$
$= 315°$

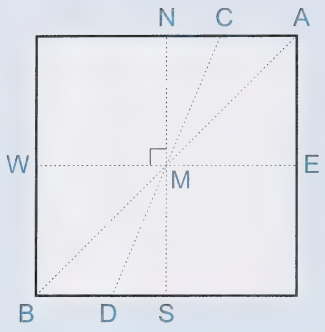
f. $\angle NMW = 360° - 90°$
$= 270°$ (So are $\angle NME$, $\angle EMS$, and $\angle SMW$.)

$\angle NMB = 180° + 45°$
$= 225°$ (So are $\angle WMA$, $\angle SMA$, and $\angle EMB$.)

$\angle WMB = 360° - 45°$
$= 315°$ (So is $\angle NMA$.)

g. Fold $\angle NMA$ in half. Each half will be $45° \div 2 = 22.5°$.

h.



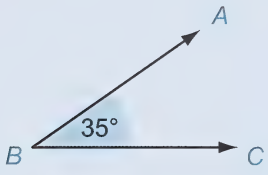
In the diagram, $\angle CME = 22.5° + 45°$

$= 67.5°$

$\angle WMD$ is also $67.5°$.

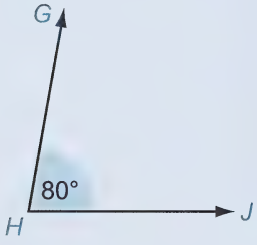
2. Textbook, page 301, "Put into Practice," questions 1, 2, 3.a., and 3.b.

1. a. acute

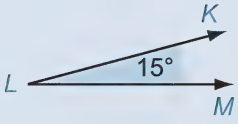


b. obtuse

c. acute

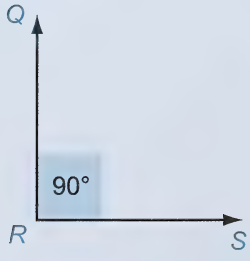


d. acute

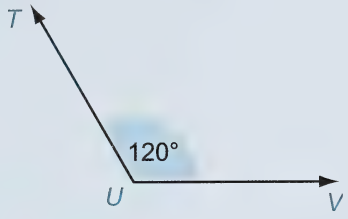


e. reflex

f. right



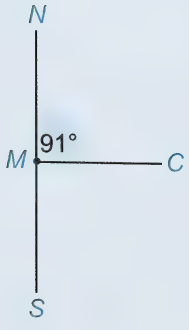
g. obtuse



3. a.

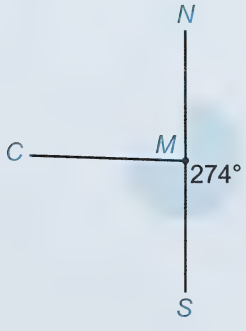
b. i.

ii.



iii.

iv.



Lesson 2: Angle Relationships

1. Textbook, pages 302 and 303, "Investigation," questions 1 to 3

1. a. 90° or 270°
b. 180°
c. 45° or 315°
d. 45° or 315°
e. 225° or 135°

2. $a = 90° \div 2$
$= 45°$

Each angle is $45°$.

3. $a = b = 45°$

Another pair of opposite angles is the angle between west and northwest and the angle between east and southeast.

Another pair of adjacent angles is the angle between west and northwest and the angle between north and northwest.

2. Textbook, page 304, "Investigation," questions 1 to 3

1. As $\angle PEL$ gets larger, $\angle CEN$ gets larger.
As $\angle PEL$ gets smaller, $\angle CEN$ gets smaller.
As $\angle PEC$ gets smaller, $\angle LEN$ gets smaller.
As $\angle PEC$ gets larger, $\angle LEN$ gets larger.

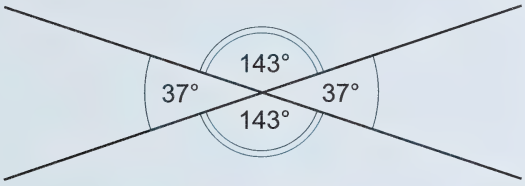
$\angle PEL$ and $\angle CEN$ get larger and smaller together.
$\angle PEC$ and $\angle LEN$ get larger and smaller together.

2. a. The other pair of opposite angles is $\angle PEC$ and $\angle LEN$.

b. $\angle PEL = 55°$
$\angle CEN = 55°$
$\angle PEC = 125°$
$\angle LEN = 125°$

c. $\angle PEL = \angle CEN$
$\angle PEC = \angle LEN$

d. Answers will vary. A sample answer is given.



e. The sizes of opposite angles are equal.

3. a. i. $\angle PEL + \angle LEN = 55° + 125°$
$= 180°$

ii. $\angle CEN + \angle LEN = 55° + 125°$
$= 180°$

iii. $\angle CEN + \angle PEC = 55° + 125°$
$= 180°$

iv. $\angle PEL + \angle PEC = 55° + 125°$
$= 180°$

b. The sum of the measures of each pair of angles is $180°$.

c. The angles formed by each pair of angles are straight angles.

$\angle PEL + \angle LEN = \angle PEN$ $\quad$ $\angle CEN + \angle PEC = \angle PEN$
$\angle CEN + \angle LEN = \angle CEL$ $\quad$ $\angle PEL + \angle PEC = \angle CEL$

3. Textbook, page 306, "Put into Practice," questions 1 to 3

1. a. $\angle AMD$ and $\angle BMC$ are equal opposite angles.
$\angle AMB$ and $\angle CMD$ are equal opposite angles.

b. $\angle AMB + \angle AMD = 180°$
$121° + \angle AMD = 180°$
$\angle AMD = 180° - 121°$
$= 59°$

$\angle CMD = \angle AMB$
$= 121°$

$\angle BMC = \angle AMD$
$= 59°$

2. Boris turns through 130° in order to head home. Cheriane turns through 50° in order to head home.

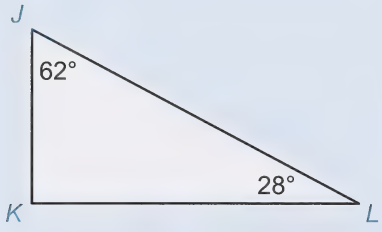
3. **a.** $a + 23° = 90°$
$a = 90° - 23°$
$= 67°$

b. $c + 32° = 90°$
$c = 90° - 32°$
$= 58°$

$d = 90°$ (opposite a right angle)

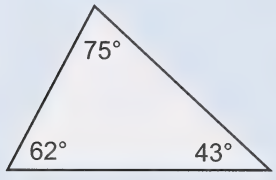
4. **Textbook, page 305, "Investigation," question 1**

1. **a.** Answers will vary. A sample answer is given.



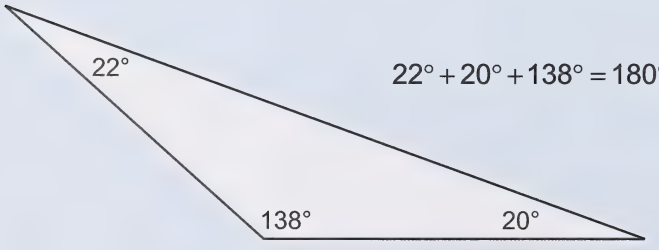
$90° + 62° + 28° = 180°$

b. Answers will vary. A sample answer is given.



$62° + 75° + 43° = 180°$

c. Answers will vary. A sample answer is given.



$22° + 20° + 138° = 180°$

5. Textbook, page 307, "Investigation," questions 4 and 5

4. a. $65° + 80° + e = 180°$

$145° + e = 180°$

$e = 180° - 145°$

$= 35°$

$80° + f = 180°$

$f = 180° - 80°$

$= 100°$

b. $e = 58°$ opposite angles

$90° + e + f = 180°$

$90° + 58° + f = 180°$

$148° + f = 180°$

$f = 180° - 148°$

$= 32°$

$f + g = 180°$

$32° + g = 180°$

$g = 180° - 32°$

$= 148°$

c. $65° + i + 20° = 180°$

$i + 85° = 180°$

$i = 180° - 85°$

$= 95°$

5. a. i. Half of $360°$ is $360° \div 2 = 180°$.

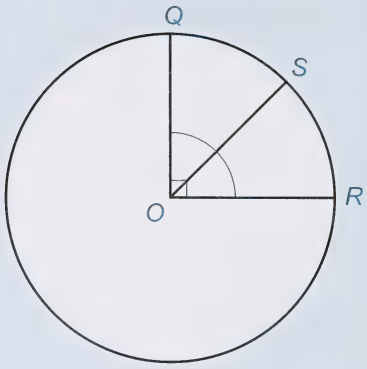
ii. Three quarters of $360°$ is $360° - 90° = 270°$.

b. $\angle QOR = 360° - 90°$

$= 270°$

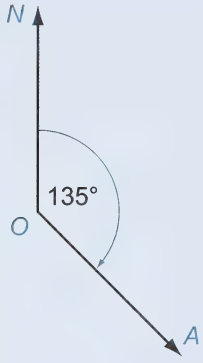
c.

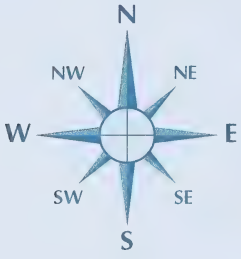
$\angle QOS$ is one eighth of 360° or $360° \div 8 = 45°$.
or $\angle QOS$ is one half of $\angle QOR$ or $90° \div 2 = 45°$.



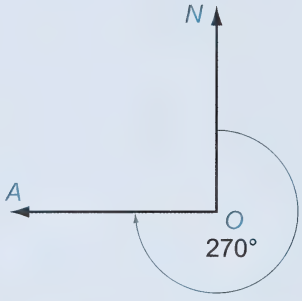
Lesson 3: Angles in Navigation

1. a. A bearing of 135° is SE.

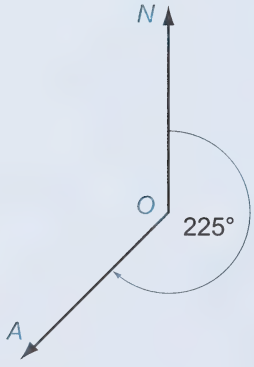




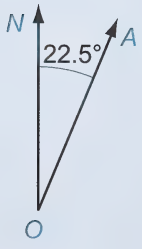
b. A bearing of 270° is W.



c. A bearing of 225° is SW.



d. A bearing of 22.5° is NNE (midway between N and NE).



2. Textbook, pages 308 and 309, "Put into Practice," questions 1 to 3

1. a. Your estimates may vary. Sample answers are given.

The bearing of OA is about $80°$.
The bearing of OB is about $260°$.

b. The bearing of OA is $80°$.
The bearing of OB is $260°$.

2. a. Your estimates may vary. Sample answers are given.

The bearing of OC is about $170°$.
The bearing of OD is about $350°$.

b. The bearing of OC is $170°$.
The bearing of OD is $350°$.

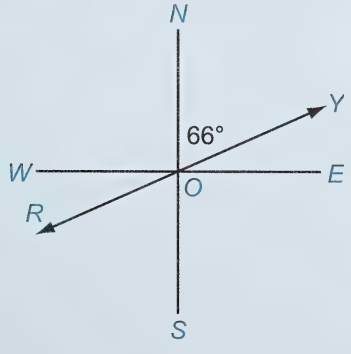
3. a. Your estimates may vary. Sample answers are given.

The bearing of OF is about $100°$.
The bearing of OG is about $280°$.

b. The bearing of OF is $100°$.
The bearing of OG is $280°$.

3. Textbook, pages 310 and 311, "Put into Practice," questions 4.a., 4.d., 5, and 6

4. a.



$\angle NOY = 66°$. This rounds to $70°$.

$70 \div 10 = 7$

The runway along OY is Runway 7.

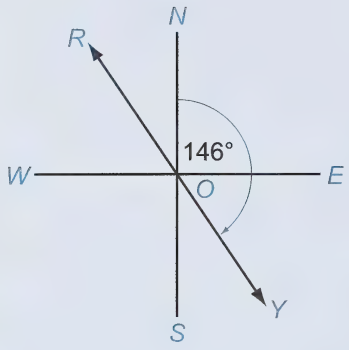
$\angle NOR = 180° + 66°$
$= 246°$

This rounds to $250°$.

$250 \div 10 = 25$

The runway along OR is Runway 25.

d.



$\angle NOY = 146°$. This rounds to $150°$.

$150 \div 10 = 15$

The runway along OY is Runway 15.

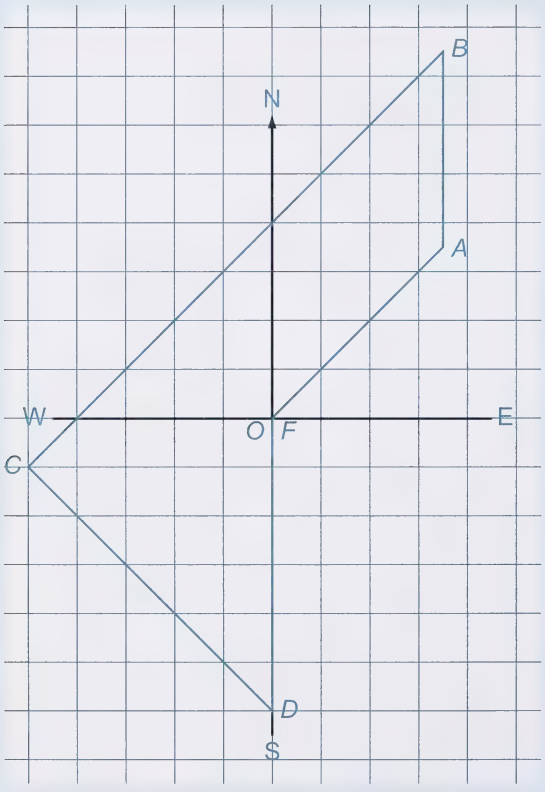
$\angle NOR = 180° + 146°$
$= 326°$

This rounds to $330°$.

$330 \div 10 = 33$

The runway along OR is Runway 33.

5. a. to e.



f. 1 cm represents 200 m.

$$OA + AB + BC + CD + DF = 5 \text{ cm} + 4 \text{ cm} + 12 \text{ cm} + 7 \text{ cm} + 6 \text{ cm}$$
$$= 34 \text{ cm}$$

34 cm represents $34 \times 200 \text{ m} = 6800 \text{ m}$.

You hiked 6800 m.

6. a. to f. Let 1 cm represent 200 m.

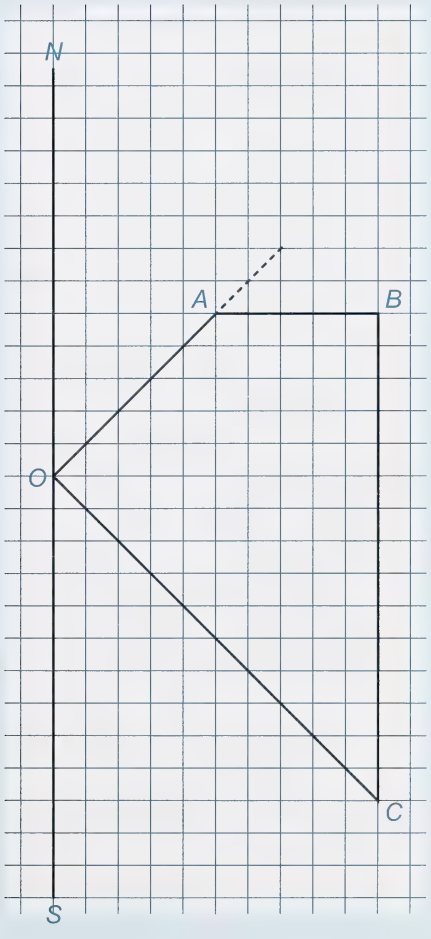
Note: The diagram is not drawn to scale.

$$\therefore OA = 700 \div 200$$
$$= 3.5 \text{ cm}$$

$$AB = 500 \div 200$$
$$= 2.5 \text{ cm}$$

$$BC = 1500 \div 200$$
$$= 7.5 \text{ cm}$$

$$CO = 1400 \div 200$$
$$= 7.0 \text{ cm}$$



Section 2: Circles

Lesson 1: Circumference of Circles

1. Textbook, page 312, "Put into Practice," questions 1 to 3

1. The length of a diameter of a circle is twice the length of a radius.

2. a.
$$d = 2 \times r$$
$$12\,680 = 2 \times r$$
$$r = 12\,680 \div 2$$
$$= 6340$$

The distance from the equator to the centre of the Earth is about 6340 km.

b.
$$d = 2 \times r$$
$$12\,640 = 2 \times r$$
$$r = 12\,640 \div 2$$
$$= 6320$$

The distance from the North Pole to the centre of the Earth is about 6320 km.

c. The shape of the Earth bulges slightly at the equator.

3.
$$d = 2 \times r$$
$$= 2 \times 1500$$
$$= 3000$$

The diameter of the circle that shows the bees' territory is 3000 m.

2. Textbook, page 313, "Investigation," questions 1 to 3

1. a. You need just over three twist ties to fit around the circumference.

b. The circumference of a circle is a little more than three times the length of the diameter.

2. You can calculate the approximate value of the circumference of a circle by multiplying the length of the diameter by 3.

3. $C \doteq 3 \times d$

$\doteq 3 \times 6$

$\doteq 18$

The approximate value of the circumference is 18 cm.

3. Textbook, page 314, "Investigation," questions a. to e.

a. to. e. Answers will vary. In each case, $C \div d \doteq 3.14$, which is close to π.

4. Textbook, pages 316 to 318, "Put into Practice," questions 4 to 8 and question 10

4. a. The circumference of the base of the container will be larger than the height of the container.

b. Answers will vary.

c. i. height $= 3 \times d$

$= 3 \times 6$

$= 18$

The height of the container is 18 cm.

ii. $C = \pi d$

$= \pi \times 6$

$\doteq 18.849\,555\,92$

The circumference of the base of the container is about 18.8 cm.

iii. The circumference of the base is about 0.8 cm longer than the height of the container.

5. a. The length of the second hand is equal to the radius of the circle that its tip sweeps out in 1 min.

$d = 2r$

$= 2 \times 4.1$

$= 8.2$

The diameter of the circle is about 8.2 cm.

$$C = \pi d$$
$$= \pi \times 8.2$$
$$\doteq 25.761\,059\,76$$

The tip of the second hand travels about 26 cm in 1 min.

b. $2\text{ h} = 2 \times 60\text{ min}$
$= 120\text{ min}$

The tip of the second hand travels about $120 \times C$ in 2 h.

$$120 \times C = 120 \times \pi \times d$$
$$= 120 \times \pi \times 8.2$$
$$\doteq 3091.327\,171\text{ cm}$$
$$\doteq 30.913\,271\,71\text{ m}$$

The tip of the second hand travels about 31 m in 2 h.

6. a. $d = 2r$
$= 2 \times 2.9$
$= 5.8$

The diameter of the inside circle is 5.8 m.

$$C = \pi d$$
$$= \pi \times 5.8$$
$$\doteq 18.221\,237\,39$$

$$\text{distance travelled in 50 rotations} = 50C$$
$$\doteq 50 \times 18.221\,237\,39$$
$$\doteq 911.061\,869\,5$$

You would travel about 911 m when you stand on the inside edge.

b. radius of outside circle $= 2.9 + 2.7$
$= 5.6$ m

Therefore, $d = 2r$
$= 2 \times 5.6$
$= 11.2$

The diameter of the outside circle is 11.2 m.

circumference of outside circle $= \pi d$
$= \pi \times 11.2$
$\doteq 35.185\,837\,72$

distance travelled in 50 rotations $= 50C$
$\doteq 50 \times 35.185\,837\,72$
$\doteq 1759.291\,886$

You would travel about 1759 m when you stand on the outside edge.

c. cost per metre $\doteq 5000 / 1759.291\,886$
$\doteq 2.842\,052\,55$

The cost per metre when you stand on the outside is about 2.84 lira.

7. a. distance $=$ 20 rotations
$= 20C$
$= 20 \times \pi \times d$
$= 20 \times \pi \times 36$
$\doteq 2261.946\,711$ cm
$\doteq 22.619\,467\,11$ m

In 20 rotations, the wheel rolls about 23 m.

b. $1\text{ km} = 1000\text{ m}$

$= 100\,000\text{ cm}$

Divide this distance by the circumference of the wheel to determine the number of rotations in 1 km.

$$\frac{100\,000}{C} = \frac{100\,000}{\pi d}$$
$$= \frac{100\,000}{\pi \times 36}$$
$$\doteq 884.194\,128\,3$$

The trailer wheel rotates about 884 times in 1 km.

c. $1\text{ km} = 100\,000\text{ cm}$

Divide this distance by the circumference of the car tire.

$$\frac{100\,000}{C} = \frac{100\,000}{\pi d}$$
$$= \frac{100\,000}{\pi \times 60}$$
$$\doteq 530.516\,477$$

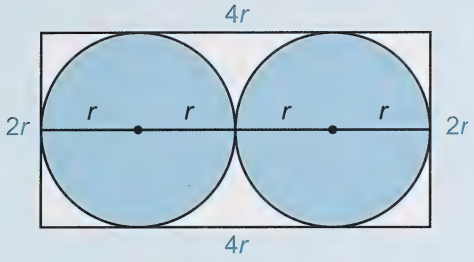
The car wheel rotates about 531 times in 1 km.

d. The trailer tire rotates faster. Compare their diameters.

$$\frac{60\text{ cm}}{36\text{ cm}} = \frac{10}{6}$$
$$= \frac{5}{3}$$
$$= 1\frac{2}{3}$$

The trailer tire rotates $1\frac{2}{3}$ times as fast as the car tire.

8. a.



$$\begin{aligned}\text{length of the rectangle} &= 2 \times d \\ &= 2 \times 2r \\ &= 4r \\ \text{width of the rectangle} &= 2r\end{aligned}$$

$$\begin{aligned}P &= 4r + 2r + 4r + 2r \\ P &= 12r \\ 60 &= 12r \\ 12r &= 60 \\ r &= 60 \div 12 \\ &= 5\end{aligned}$$

The radius of each circle is 5 cm.

b. $$\begin{aligned}d &= 2r \\ &= 2 \times 5 \\ &= 10\end{aligned}$$

$$\begin{aligned}C &= \pi d \\ &= \pi \times 10 \\ &\doteq 31.415\ 926\ 54\end{aligned}$$

The circumference of each circle is about 31.4 cm.

10. a. $$\begin{aligned}C &= \pi d \\ &= \pi \times 12\ 680 \\ &\doteq 39\ 835.394\ 85\end{aligned}$$

The stone travels about 39 835 km in one day.

b. $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$$\doteq \frac{39\,835.394\,85 \text{ km}}{24 \text{ h}}$$

$$\doteq 1659.808\,119 \text{ km/h}$$

The stone is travelling at about 1660 km/h.

c. The bush travels about the same distance as the stone, or about 39 835 km.

Lesson 2: Area of a Circle

1. Textbook, page 319, "Investigation," questions 1 and 2

1. a. $2 \times 4 = 8$

Each side of the square is 8 cm in length.

b. $\text{area} = 8 \times 8$

$= 64$

The area of the square is 64 cm^2.

c. The area of the circle will be a little less than 64 cm^2.

2. a. The diagram is the same as the diagram on page 319 of your textbook.

b. An estimate of the area of the circle is 50 cm^2.

The area is about 50 cm^2.

2. Textbook, pages 321 and 322, "Put into Practice," questions 1 to 5

1. $A = \pi r^2$

$= \pi \times 93^2$

$\doteq 27\,171.634\,86$

The area that can be irrigated is about $27\,172 \text{ m}^2$.

2. a. $A = \pi r^2$

$= \pi \times 100^2$

$\doteq 31\,415.926\,54$

The area of a circle with a radius of 100 km is about $31\,416\ \text{km}^2$.

b. to f. Answers will depend on your location.

g. The area of the 100-km circle is approximately the area of Prince Edward Island.

3. a. If you know the radius, square the radius and multiply by π to find the area of the circle.

b. If you know the diameter, first divide the diameter by 2 to find the radius. Then, to find the area of the circle, square the radius and multiply by π.

4. $A = \pi r^2$

$= \pi \times 9^2$

$\doteq 254.469\,004\,9$

The area in which the horse will be able to graze is about $254\ \text{m}^2$.

5. a. area of washer = area of large circle – area of small circle

$$= \pi \times \left(\frac{4.5}{2}\right)^2 - \pi \times \left(\frac{2.5}{2}\right)^2$$

$$= (\pi \times 2.25^2) - (\pi \times 1.25^2)$$

$$\doteq 10.995\,574\,29$$

The area the washer covers is about $11\ \text{cm}^2$.

b. Divide 250 cm by 4.5 cm to see how many rows of washers can be stamped out.

$$250 \div 4.5 = 55.5\overline{5}$$

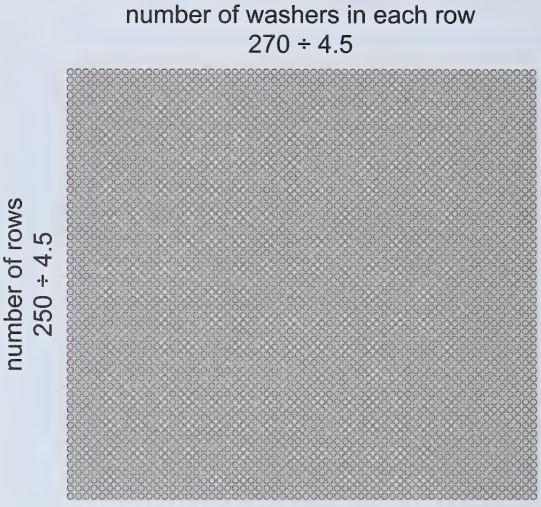
A maximum of 55 rows of washers are possible.

Divide 270 cm by 4.5 cm to see how many washers in each row can be stamped out.

$$270 \div 4.5 = 60$$

There are 60 washers possible in each row.

$$60 \times 55 = 3300$$



Therefore, 3300 washers can be stamped out of that sheet of metal.

c. **mass of 1 washer** **mass of 1800 washers**

$11\ \text{cm}^2 \times 2\ \text{g/cm}^2 = 22\ \text{g}$

$\text{mass of 1 washer} \times 1800 = 22 \times 1800$
$= 39\ 600$

The 1800 washers have a mass of 39 600 g or 39.6 kg.

No, she will not excee the 40-kg limit for luggage.

3. Textbook, pages 324 to 326, "Put into Practice," questions 7, 8, 9, 11, 12, and 13

7.

Radius (nearest 0.1 cm)	Diameter (nearest 0.1 cm)	Area (nearest cm^2)
2.9 cm	5.8 cm	26 cm^2
1.3 cm	2.6 cm	5 cm^2
5.0 cm	10 cm	78 cm^2

8. a. $A = \pi r^2$ and $A = 637.6$

$$637.6 = \pi r^2$$
$$\frac{637.6}{\pi} = \frac{\pi r^2}{\pi}$$
$$\frac{637.6}{\pi} = r^2$$
$$r^2 = \frac{637.6}{\pi}$$
$$\doteq 202.954\,383\,4$$
$$r \doteq \sqrt{202.954\,383\,4}$$
$$\doteq 14.246\,205\,93$$
$$d = 2r$$
$$\doteq 28.492\,411\,86$$

The diameter of the pizza pie plate is about 28.5 cm.

b. To find how many rows of pie plates are possible, divide 180 cm by 28.5 cm.

$$180 \div 28.5 \doteq 6.315\,789\,474$$

Therefore, 6 rows of pie plates are possible.

To find how many plates are possible in each row, divide 200 cm by 28.5 cm.

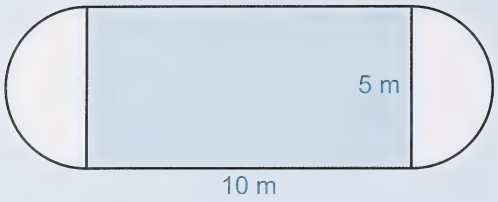
$$200 \div 28.5 \doteq 7.017\,543\,86$$

Therefore, 7 plates are possible in each row.

$$7 \times 6 = 42$$

Therefore, 42 plates can be cut from the sheet of metal.

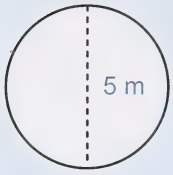
9.



$$\begin{aligned}\text{area of rectangle} &= 10 \times 5 \\ &= 50\end{aligned}$$

The area of the rectangle is $50\ \text{m}^2$.

Put the two semi-circular ends together to form a circle.



$$\begin{aligned}d &= 5 \\ r &= 5 \div 2 \\ &= 2.5\end{aligned}$$

The area of the circle is

$$\begin{aligned}A &= \pi r^2 \\ &= \pi \times 2.5^2 \\ &\doteq 19.634\ 954\ 08\end{aligned}$$

The area of the circle is about $20\ \text{m}^2$.

$$\begin{aligned}\text{area of the ice} &= \text{area of circle} + \text{area of rectangle} \\ &\doteq 20 + 50 \\ &\doteq 70\end{aligned}$$

The area of the ice surface is about $70\ \text{m}^2$.

11. $A = \pi r^2$

$$\begin{aligned}&= \pi \times \left(\frac{26}{2}\right)^2 \\ &= \pi \times 13^2 \\ &\doteq 530.929\ 158\ 5\end{aligned}$$

About $531\ \text{cm}^2$ of cloth is required for each ring.

12. Find the circumference of 1 ring and then multiply by 5 because there are 5 rings.

$$\begin{aligned}\text{total wire} &= 5C \\ &= 5 \times \pi d \\ &= 5 \times \pi \times 2 \\ &\doteq 31.415\,926\,54\end{aligned}$$

You would need about 32 cm of wire if you want the diameter of each ring to be 2 cm.

13. a. i.
$$\begin{aligned}C &= \pi d \\ &= \pi \times 8 \\ &\doteq 25.132\,741\,23\end{aligned}$$

The circumference of a medal is about 25 cm.

ii.
$$\begin{aligned}A &= \pi r^2 \\ &= \pi \times 4^2 \\ &\doteq 50.265\,482\,46\end{aligned}$$

The area of one side of a medal is about $50\ \text{cm}^2$.

b.
$$\begin{aligned}\text{volume} &= \text{area of base} \times \text{thickness} \\ &\doteq 50.265\,482\,46 \times 0.9 \\ &\doteq 45.238\,934\,21\end{aligned}$$

The volume of a medal is about $45\ \text{cm}^3$.

c.
$$\begin{aligned}\text{cost of a gold medal} &= 256\ \text{g} \times \$15.75/\text{g} \\ &= \$4032.00\end{aligned}$$

$$\begin{aligned}\text{cost of a silver medal} &= 250\ \text{g} \times \$0.25/\text{g} \\ &= \$62.50\end{aligned}$$

d. radius of a one-cent coin $= 1.9 \text{ cm} \div 2$

$= 0.95 \text{ cm}$

area of one side of a one-cent coin $= \pi r^2$

$= \pi \times 0.95^2$

$\doteq 2.835\,287\,37$

volume of a one-cent coin = area of base $\times$ thickness

$= A \times 0.145$

$\doteq 0.411 \text{ cm}^3$

Divide the volume of a bronze medal by the volume of a one-cent coin.

$45 \div 0.411 \doteq 109.489\,0511$

A bronze medal is a little over 109 times the volume of a one-cent coin.

Section 3: Pythagoras's Theorem

Lesson 1: Square Roots

1. Textbook, page 352, "Put into Practice," questions 1 to 3

1.

Number	1	2	3	4	5	6	7	8	9	10	11	12
Number^2	1	4	9	16	25	36	49	64	81	100	121	144

2. a. 4 **b.** 5 **c.** 6 **d.** 7 **e.** 12

f. 8 **g.** 10 **h.** 11 **i.** 9

3. The answers are the same as in question 2.

2. Textbook, page 353, "Put into Practice," question 4

4. a. $9 < 11 < 16$

$\sqrt{9} < \sqrt{11} < \sqrt{16}$

$3 < \sqrt{11} < 4$

$\therefore \sqrt{11} \doteq 3.3$ since 11 is closer to 9.

b. $25 < 34 < 36$

$\sqrt{25} < \sqrt{34} < \sqrt{36}$

$5 < \sqrt{34} < 6$

$\therefore \sqrt{34} \doteq 5.8$ since 34 is closer to 3

c. $49 < 50 < 64$

$\sqrt{49} < \sqrt{50} < \sqrt{64}$

$7 < \sqrt{50} < 8$

$\therefore \sqrt{50} \doteq 7.1$ since 50 is closer to 49

d. $64 < 78 < 81$

$\sqrt{64} < \sqrt{78} < \sqrt{81}$

$8 < \sqrt{78} < 9$

$\therefore \sqrt{78} \doteq 8.8$ since 78 is closer to 81.

e. $81 < 95 < 100$

$\sqrt{81} < \sqrt{95} < \sqrt{100}$

$9 < \sqrt{95} < 10$

$\therefore \sqrt{95} \doteq 9.7$ since 95 is closer to 100.

f. $100 < 120 < 121$

$\sqrt{100} < \sqrt{120} < \sqrt{121}$

$10 < \sqrt{120} < 11$

$\therefore \sqrt{120} \doteq 10.9$ since 120 is closer to 1

3. Textbook, page 353, "Put into Practice," questions 5 and 6

5. a. $\sqrt{11} \doteq 3.317$ **b.** $\sqrt{34} \doteq 5.831$ **c.** $\sqrt{50} \doteq 7.071$

d. $\sqrt{78} \doteq 8.832$ **e.** $\sqrt{95} \doteq 9.747$ **f.** $\sqrt{120} \doteq 10.954$

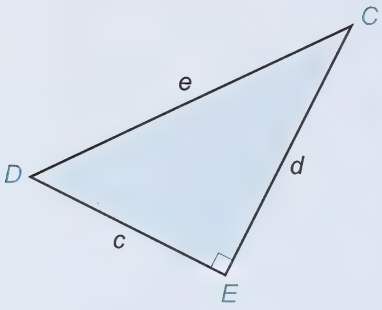
6. a. $\sqrt{18} \doteq 4.242\,640\,7$ **b.** $\sqrt{50} \doteq 7.071\,067\,8$

c. $\sqrt{86} \doteq 9.273\,618\,5$ **d.** $\sqrt{95} \doteq 9.746\,794\,3$

Lesson 2: Pythagoras's Theorem

Textbook, page 354, "Investigation," questions 1 and 2

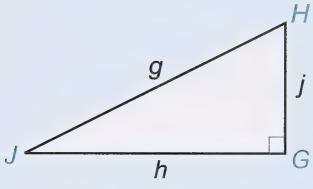
1.



The hypotenuse is *CD*, the longest side.

$$CD^2 = DE^2 + CE^2 \qquad \text{or} \qquad e^2 = c^2 + d^2$$

2.



Because $g^2 = h^2 + j^2$, the longest side is side *g*.

Lesson 3: Applying Pythagoras's Theorem

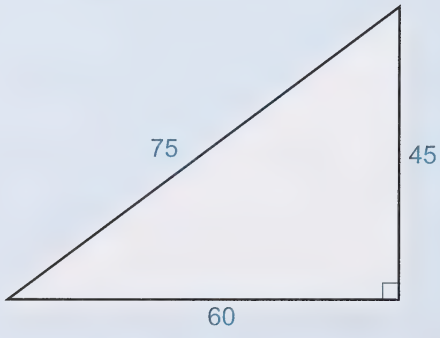
1. a.

$$75^2 = 5625$$

$$45^2 + 60^2 = 2025 + 3600$$

$$= 5625$$

$$\therefore 75^2 = 45^2 + 60^2$$

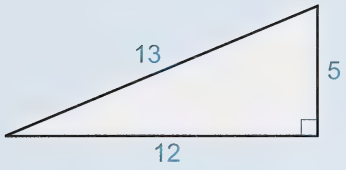


b. $13^2 = 169$

$5^2 + 12^2 = 25 + 144$

$= 169$

$\therefore 13^2 = 5^2 + 12^2$

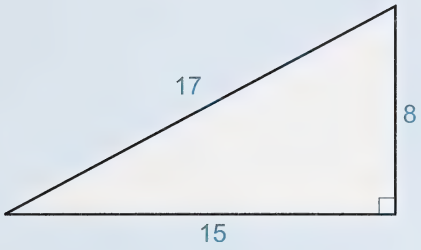


c. $17^2 = 289$

$8^2 + 15^2 = 64 + 225$

$= 289$

$\therefore 17^2 = 8^2 + 15^2$



2. Textbook, pages 355 to 358, "Put into Practice," questions 1 to 6

1. $c^2 = 9 + 16$

$= 25$

2. $25 = 9 + r^2$

$9 + r^2 = 25$

$r^2 = 25 - 9$

$= 16$

3. The hypotenuse is 5 units for each of the right-angle triangles.

4. $AJ^2 = AC^2 + CJ^2$

$c^2 = j^2 + a^2$

$c^2 = 56^2 + 90^2$

$c^2 = 11\,236$

$c = \sqrt{11\,236}$

$c = 106$

5.
$$AB^2 = BK^2 + AK^2$$
$$k^2 = a^2 + b^2$$
$$10^2 = a^2 + 8^2$$
$$100 = a^2 + 64$$
$$a^2 + 64 = 100$$
$$a^2 = 100 - 64$$
$$a^2 = 36$$
$$a = \sqrt{36}$$
$$a = 6$$

The length of the missing side is 6 cm.

6. a.
$$EM^2 = DE^2 + DM^2$$
$$d^2 = m^2 + e^2$$
$$d^2 = 30^2 + 50^2$$
$$d^2 = 900 + 2500$$
$$d^2 = 3400$$
$$d = \sqrt{3400}$$
$$\doteq 58.309\ 518\ 95$$

The length of the missing side is about 58.3 cm.

b.
$$NS^2 = NF^2 + FS^2$$
$$f^2 = s^2 + n^2$$
$$159^2 = 84^2 + n^2$$
$$25\ 281 = 7056 + n^2$$
$$7056 + n^2 = 25\ 281$$
$$n^2 = 25\ 281 - 7056$$
$$n^2 = 18\ 225$$
$$n = \sqrt{18\ 225}$$
$$n = 135$$

The length of the missing side is 135 cm.

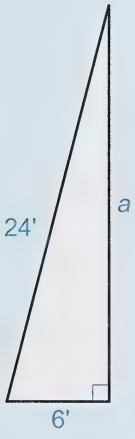
c.

$$PG^2 = PA^2 + AG^2$$
$$a^2 = g^2 + p^2$$
$$75^2 = 21^2 + p^2$$
$$5625 = 441 + p^2$$
$$441 + p^2 = 5625$$
$$p^2 = 5625 - 441$$
$$p^2 = 5184$$
$$p = \sqrt{5184}$$
$$p = 72$$

The length of the missing side is 72 cm.

3. Textbook, pages 358 to 360, "Put into Practice," questions 7 to 13

7. a.

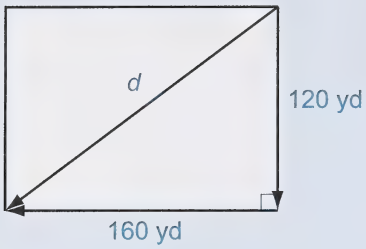


b.

$$24^2 = 6^2 + a^2$$
$$576 = 36 + a^2$$
$$36 + a^2 = 576$$
$$a^2 = 576 - 36$$
$$a^2 = 540$$
$$a = \sqrt{540}$$
$$\doteq 23.237\ 900\ 08$$

The ladder reaches about 23.2 ft up the wall.

8. a.



b. $d^2 = 120^2 + 160^2$

$d^2 = 14\,400 + 25\,600$

$d^2 = 40\,000$

$d = \sqrt{40\,000}$

$d = 200$

She would walk 200 yd along the diagonal.

She jogged $120 \text{ yd} + 160 \text{ yd} = 280 \text{ yd}$ altogether.

The shorter route is along the diagonal.

c. $280 - 200 = 80$

The diagonal distance is 80 yd shorter.

9. Answers will vary depending on your personal circumstances.

10. $c^2 = 200^2 + 150^2$

$c^2 = 40\,000 + 22\,500$

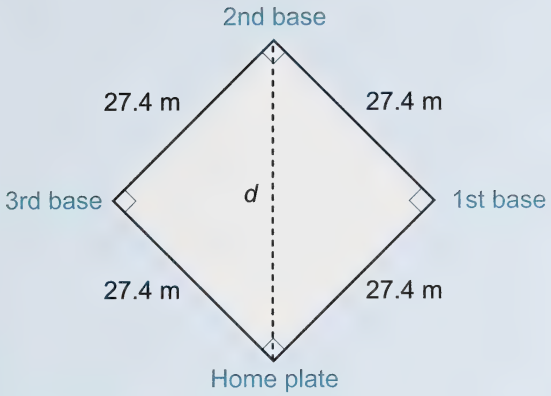
$c^2 = 62\,500$

$c = \sqrt{62\,500}$

$c = 250$

The straight-line distance is 250 yd.

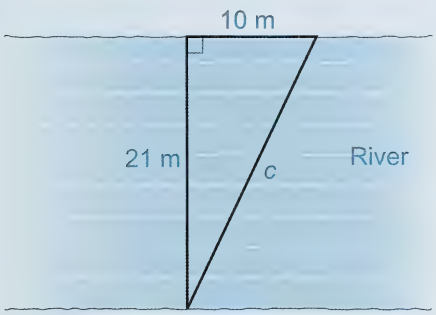
11.



$$d^2 = 27.4^2 + 27.4^2$$
$$d^2 = 750.76 + 750.76$$
$$d^2 = 1501.52$$
$$d = \sqrt{1501.52}$$
$$d \doteq 38.749\,451\,61$$

The length of the throw to second base from home plate is about 38.7 m.

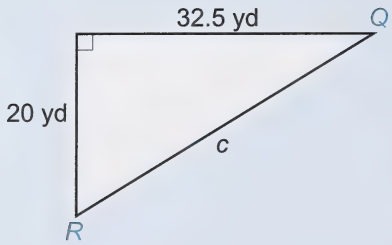
12. a.



b. $c^2 = 10^2 + 21^2$
$c^2 = 100 + 441$
$c^2 = 541$
$c = \sqrt{541}$
$c \doteq 23.259\,406\,7$

The swimmer travelled about 23 m.

13.



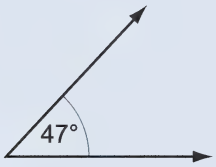
$$c^2 = 20^2 + 32.5^2$$
$$c^2 = 400 + 1056.25$$
$$c^2 = 1456.25$$
$$c = \sqrt{1456.25}$$
$$c \doteq 38.160\,843\,81$$

The receiver is about 38 yd from the quarterback.

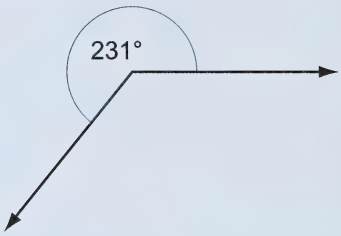
Review

1. Textbook, pages 327 to 329, "Review," questions 1 to 9

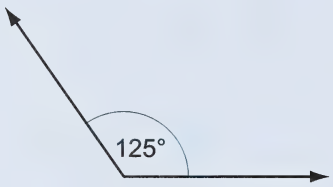
1. a. acute angle



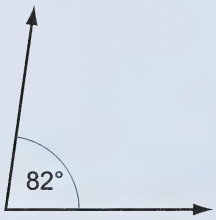
b. reflex angle



c. obtuse angle



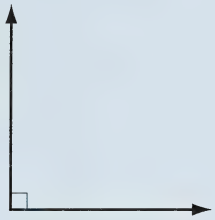
d. acute angle



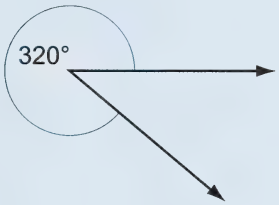
e. obtuse angle

f. acute angle

g. right angle



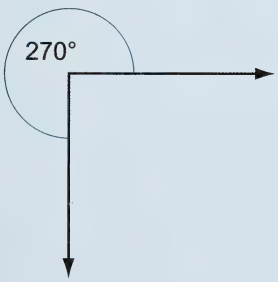
h. reflex angle



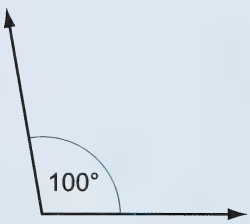
i. straight angle

j. obtuse angle

k. reflex angle



l. obtuse angle



2. a. $m + 42° + 90° = 180°$ ← triangle sum

$m + 132° = 180°$

$m = 180° - 132°$

$m = 48°$

b. $n = 70°$ ← opposite angle

$p + 70° = 180°$ ← straight angle

$p = 180° - 70°$

$p = 110°$

c. $q + 80° = 90°$ ← right angle

$q = 90° - 80°$

$q = 10°$

$r = 80°$ ← opposite angle

d. $t + 130° = 180°$ ← straight angle

$t = 180° - 130°$

$t = 50°$

$u + 60° + 50° = 180°$ ← triangle sum

$u + 110° = 180°$

$u = 180° - 110°$

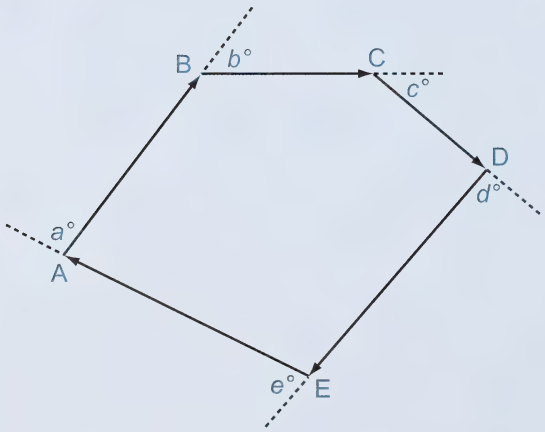
$u = 70°$

$v + 70° = 180°$ ← straight angle

$v = 180° - 70°$

$v = 110°$

3. a.



b. $b° = 50°$

$c° = 40°$

$d° = 90°$

$e° = 75°$

$b° + c° + d° + e° = 50° + 40° + 90° + 75°$

$= 255°$

c. $a° = 105°$

$a° + b° + c° + d° + e° = 360°$

d. The sum of the angles will be $360°$ even though individual angles may differ.

e. A complete turn is $360°$. Sarah turned around once in following her course.

4. a. $C = \pi d$

$= \pi \times 6$

$\doteq 18.849\,555\,92$

Rounded to 2 decimal places, the circumference is 18.85 cm.

b. $r = 12$

$d = 2r$

$= 24$

$C = \pi d$

$= \pi \times 24$

$\doteq 75.398\,223\,69$

Rounded to 2 decimal places, the circumference is 75.40 m.

c. $r = 32$

$d = 2r$

$= 64$

$C = \pi d$

$= \pi \times 64$

$\doteq 201.061\,929\,8$

Rounded to 2 decimal places, the circumference is 201.06 mm.

d. $C = \pi d$

$= \pi \times 15$

$\doteq 47.123\,889\,8$

Rounded to 2 decimal places, the circumference is 47.12 cm.

5. $C = \pi d$ and $C = 50.24$ cm

$50.24 = \pi \times d$

$d = \dfrac{50.24}{\pi}$

$\doteq 15.991\,888\,68$

$r = d \div 2$

$\doteq 15.991\,888\,68 \div 2$

$\doteq 7.995\,944\,341$

Rounded to 1 decimal place, the diameter is 16.0 cm and the radius is 8.0 cm.

6. $A = \pi r^2$ and $A = 153.86\ \text{m}^2$

$153.86 = \pi r^2$

$\frac{153.86}{\pi} = r^2$

$r^2 \doteq 48.975\ 159\ 09$

$r \doteq \sqrt{48.975\ 159\ 09}$

$\doteq 6.998\ 225\ 424$

$d = 14.0\ \text{m}$

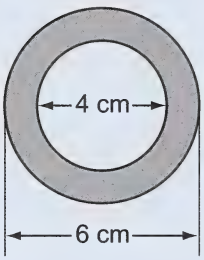
$d = 2r$

$\doteq 2 \times 6.998\ 225\ 424$

$\doteq 13.996\ 450\ 85$

Rounded to 1 decimal place, the radius is 7.0 m and the diameter is 14.0 m.

7.



inside radius $= 4 \div 2$

$= 2$

outside radius $= 6 \div 2$

$= 3$

area of metal = area of outside circle – area of inside circle

$= (\pi \times 3^2) - (\pi \times 2^2)$

$\doteq 15.707\ 963\ 27$

The area of metal in the washer is about $15.71\ \text{cm}^2$.

8. $C = 722.5\ \text{m}$

$C = \pi d$

$\therefore\ \pi d = 722.5$

$d = \frac{722.5}{\pi}$

$\doteq 229.978\ 892\ 8$

$r = d \div 2$

$\doteq 114.989\ 446\ 4$

$$A = \pi r^2$$
$$\doteq \pi \times (114.989\,446\,4)^2$$
$$\doteq 41\,539.937\,52$$

The area irrigated is about $41\,540\ m^2$.

9. $d = 2r$
$= 2 \times 6$
$= 12$

$C = \pi d$
$= \pi \times 12$
$\doteq 37.699\,111\,84$

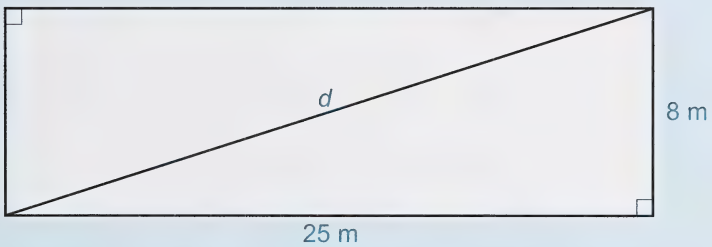
The circumference of the tipi is about 38 m.

$$A = \pi r^2$$
$$= \pi \times 6^2$$
$$\doteq 113.097\,335\,5$$

The area of the tipi is about $113\ m^2$.

2. Textbook, pages 362 and 363, "Review," questions 8 to 12

8.



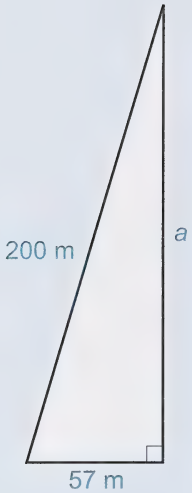
$d^2 = 8^2 + 25^2$
$d^2 = 64 + 625$
$d^2 = 689$
$d = \sqrt{689}$
$d \doteq 26.248\,809\,5$

Mike swam 262 m.

Mike swam this distance 10 times (5 times each way).

$10d \doteq 10 \times 26.248\,809\,5$
$\doteq 262.488\,095$

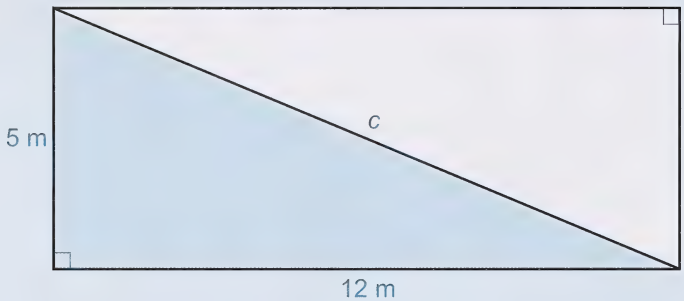
9.



$$a^2 + 57^2 = 200^2$$
$$a^2 + 3249 = 40\,000$$
$$a^2 = 40\,000 - 3249$$
$$a^2 = 36\,751$$
$$a = \sqrt{36\,751}$$
$$a \doteq 191.705\,503\,3$$

The kite is about 192 m above Henri.

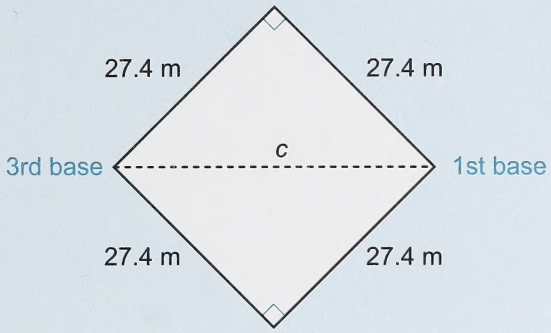
10.



$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144$$
$$c^2 = 169$$
$$c = \sqrt{169}$$
$$c = 13$$

Jill and Jean are 13 m apart.

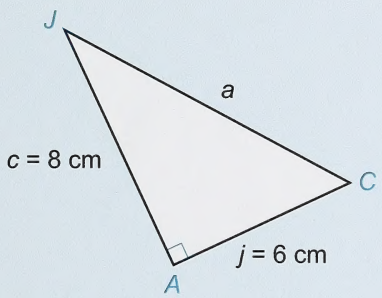
11.



$$c^2 = 27.4^2 + 27.4^2$$
$$c^2 = 750.76 + 750.76$$
$$c^2 = 1501.52$$
$$c = \sqrt{1501.52}$$
$$c \doteq 38.749\,451\,61$$

The distance between third base and first base is about 39 m.

12.



a. $JC^2 = AJ^2 + AC^2$

$a^2 = c^2 + j^2$

$a^2 = 8^2 + 6^2$

$a^2 = 64 + 36$

$a^2 = 100$

$a = \sqrt{100}$

$a = 10$

$\therefore JC = 10$ cm

b. perimeter $= JC + AJ + AC$

$= a + c + j$

$= 10 + 8 + 6$

$= 24$ cm

c. area $= 0.5 \times$ base $\times$ height

$= 0.5 \times 6 \times 8$

$= 24$ cm^2

d. They are the same.

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